Notes 9

Torsional Vibrations – a (twisted) Overview

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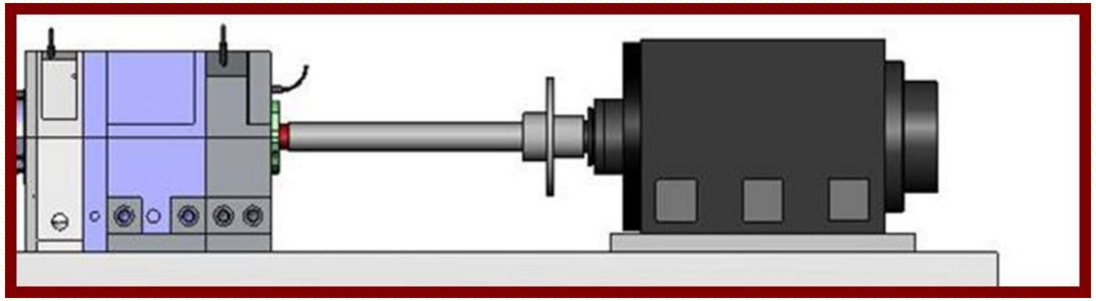
Mast-Childs Chair Professor



Torsional Vibrations – a (twisted) Overview

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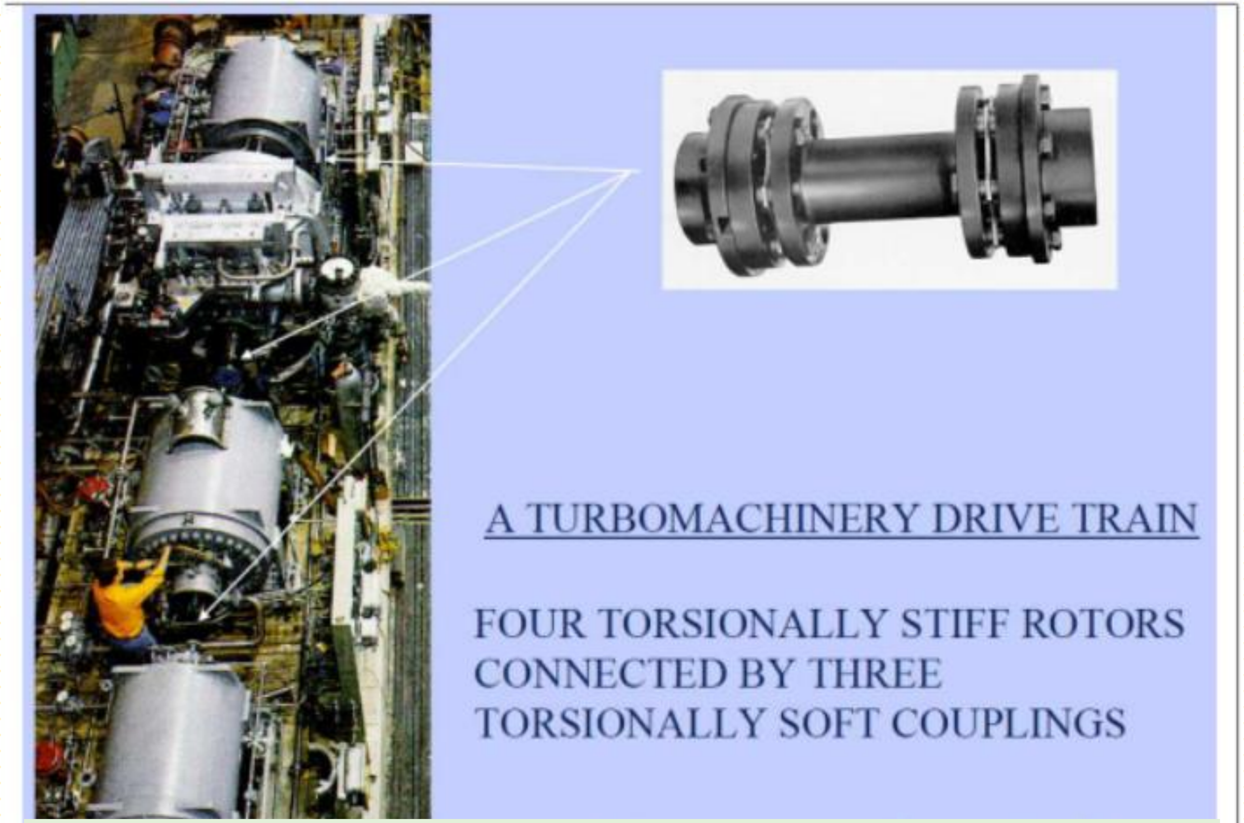
& Adolfo Delgado, Texas A&M University - Mechanical Engineering Department, January 2018

The Basics of Torsional Vibrations

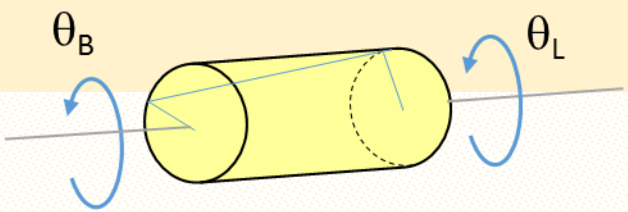


A typical high speed drive train includes a motor and a gearbox driving rotating machinery through flexible couplings.

- Torsional vibration is oscillatory twisting of the shafts in a rotor assembly that is superimposed to the running speed.
- The frequency can be externally forced, or can be an eigenvalue (natural frequency of the torsional system).
- A resonance will occur if a forcing frequency coincides with a natural frequency.
- Individual turbomachine rotors are usually stiff enough in torsion to avoid typical torsional excitation frequency range.

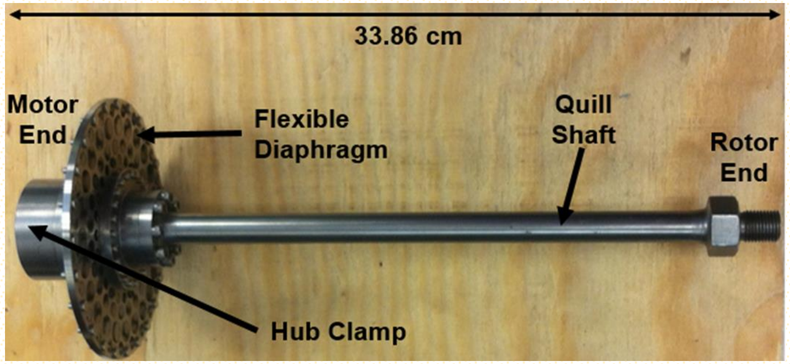
Couplings

$$T_{trans} = K_{\theta}(\theta_B - \theta_L)$$

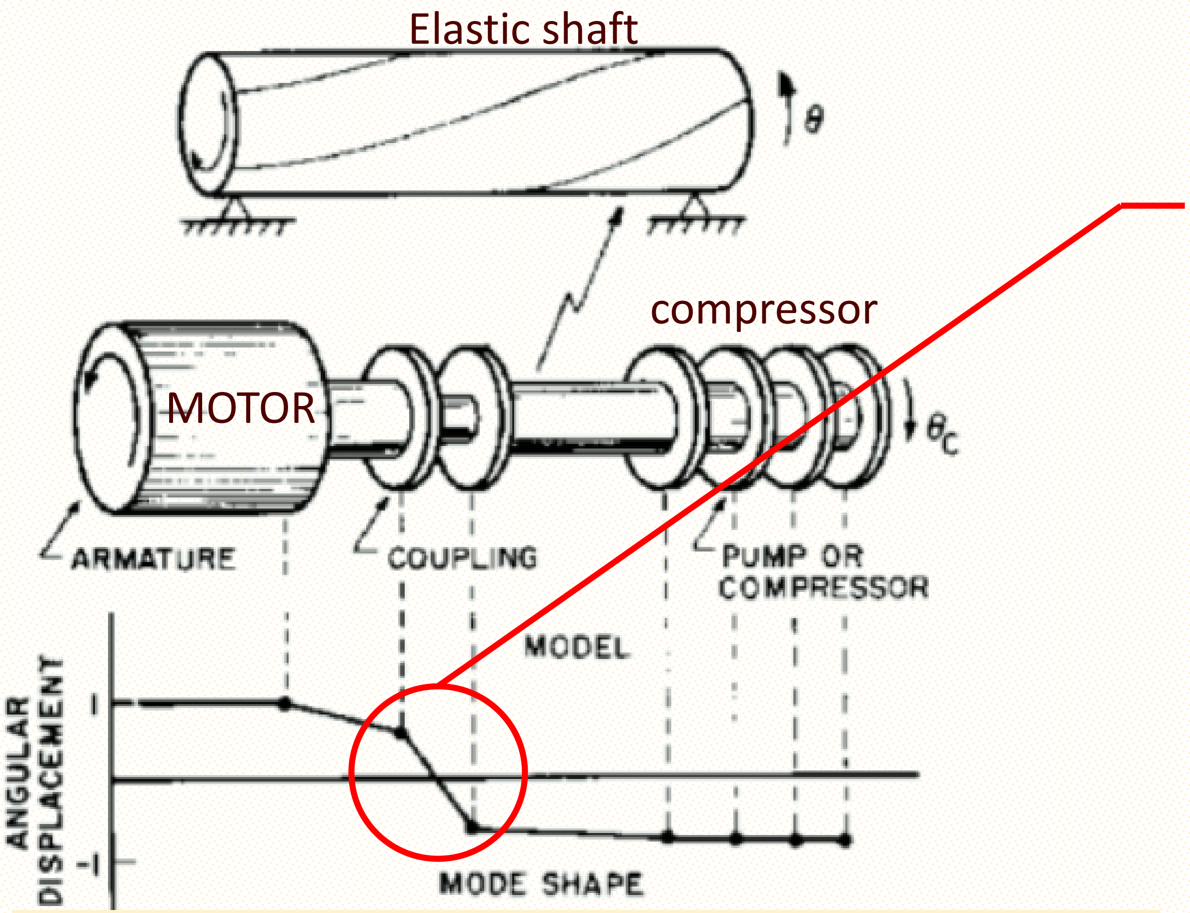


- When rotating machines are connected
  together via shafts or couplings, however, each of the individual rotors can act as a single massive inertia.
- **Couplings and connecting shafts have relatively low torsional stiffness and yield lower system natural frequencies.**
- **Torsional natural frequencies are typically low <60Hz.**

- Synchronous electric motors can produce pulsating torque at low frequency during startup.
- **Torsional vibration issues are more commonly associated with diesel engines (reciprocating ICEs) driving electric generators or marine propellers.**



The simplest torsional model



Torsional Model of a Compressor Drive Train

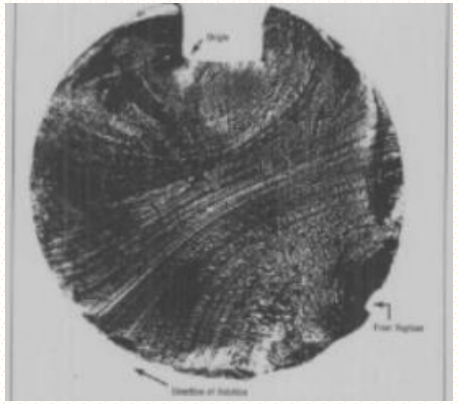
- Figure illustrates the 1st torsional elastic mode (fundamental mode)
- **The flexibility of the coupling is the dominant source of compliance**.
- Synchronous electric motors can produce pulsating torque at low frequency during startup.
- A system so simple only has a single natural frequency low enough to be excited by the most typical sources of torsional excitation.

From Eshleman, Proc. 6th Turbomachinery Symposium, 1977

Torsional vibration issues

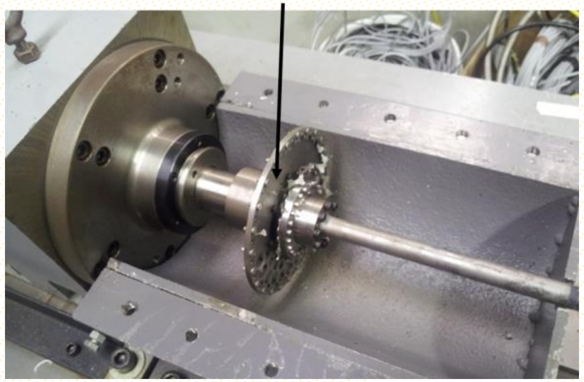
Common Indicators

- **Coupling Failures**
- Torsional shear **cracking of shaft** due to metal fatigue, usually arises in the vicinity of stress concentrations and propagate at 45^o to the shaft axis
- **Gear wear and rapid deterioration** (a few hours)of tooth surface and pitting of pitch line. May eventually result in tooth fatigue.
- **Shaft key failure** and shrink fit slippage
- **High noise level** if gears become periodically unloaded
- Poor product surface quality, rollers in steel mills, presses, etc.
- Presence of 1st torsional mode in lateral vibration signals



Fatigue Fracture of a Keyed Rotating Shaft & Broken Diaphragm

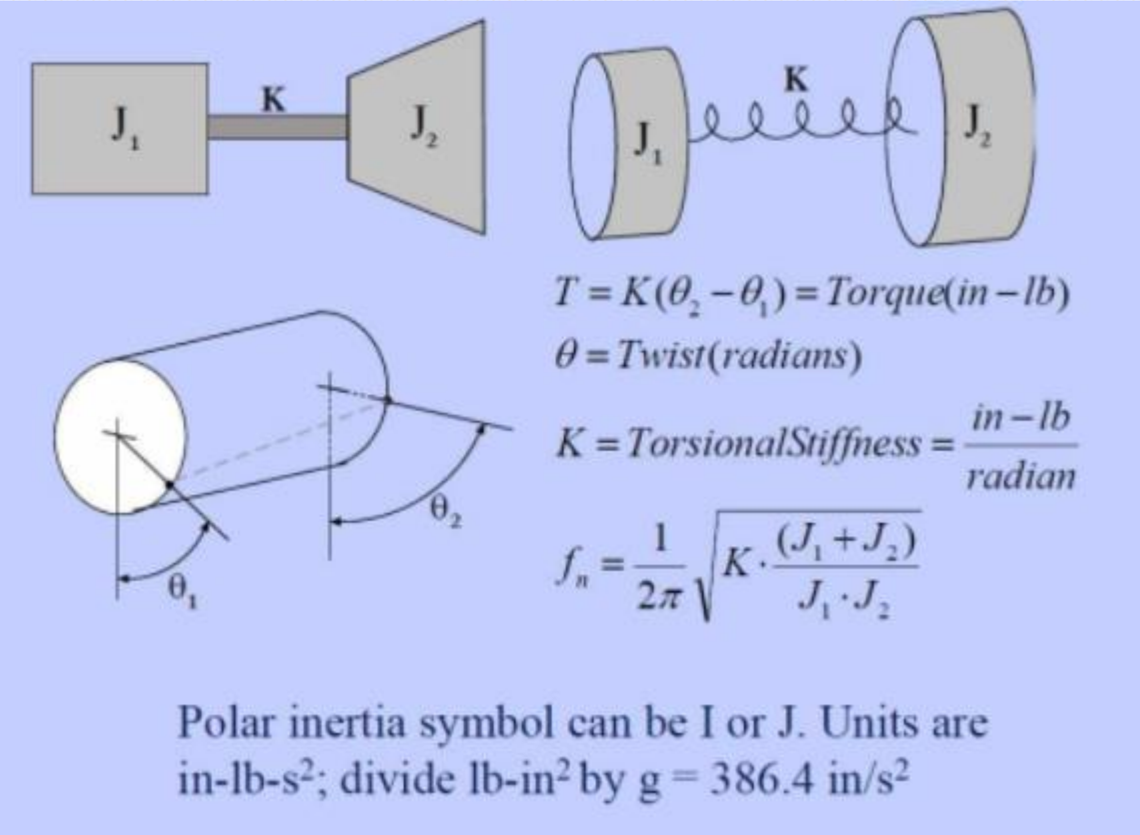
Ruptured Diaphragm



Torsional Vibrations vs. Lateral Vibrations

	Torsional	Lateral
Measurement	Requires special instrumentation, but in some instances is sensed through noise if gears are present.	Easily detected through standard instrumentation, or through vibrations transmitted to housings and foundations.
Detection	In many cases large amplitudes are not noticed until a coupling or gear fail.	Large amplitudes are noticed due to rubbing of interstage seals and turbine blades
Stresses	Always results in stress reversals with potential for fatigue failure.	In lateral synchronous vibrations, there are no stress reversals (stress is constant with circular orbits)
Natural Frequencies	Independent of shaft rotating speed.	Influenced by shaft rotating speed (gyroscopics and bearings)
Excitation	Rarely experiences instability (there are exceptions). There is no synchronous (1x) torsional excitation, except from gear pitch line runout.	Subject to self-excited vibration (instability) Most common excitation is synchronous (1X) from rotor imbalance.
Analysis	Analysis must include all rotors in the train, but each rotor can often be treated as rigid.	Analysis can usually be performed separately on each body in the train.

Fundamental Torsional Model



The most basic form of torsional model: 2 inertias connected by a single spring.

- **2 DOF → 2 nat frequencies**
- Since this model has no stiffness to ground,
  one natural frequency = zero and corresponds to unrestrained rigid body rotation of the train.
- Torsional system models of any level of complexity will in general have exactly one zero frequency mode corresponding to rigid body rotation of the entire train.

Basic Torsional Model

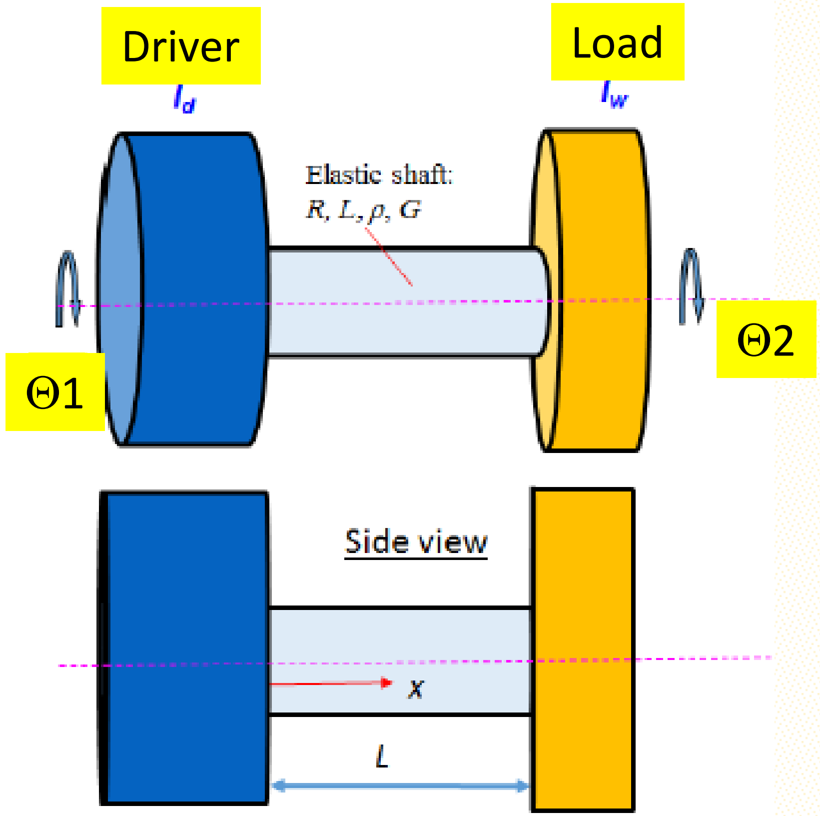
Two inertias connected by a single spring.

(b) Lumped parameter model where Id, Iw >> Is

$$\begin{pmatrix} I_d & 0 \\ 0 & I_w \end{pmatrix} \cdot \frac{d^2}{dt^2} \begin{pmatrix} \Theta_1 \\ \Theta_2 \end{pmatrix} + K_\theta \cdot \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \Theta_1 \\ \Theta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Lowest mode is rigid body with **ωn=0** and mode shape

$$\psi_1 := \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$



Graph not to scale

Find natural freqs:

Two inertias connected by a single spring.

The EOM for the two lumped inertias connected by a massless shaft of stiffness Kθ is

$$\begin{pmatrix} I_d & 0 \\ 0 & I_w \end{pmatrix} \cdot \frac{d^2}{dt^2} \begin{pmatrix} \Theta_1 \\ \Theta_2 \end{pmatrix} + K_\theta \cdot \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \Theta_1 \\ \Theta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ -0 \end{pmatrix}$$

Let $\Theta = \beta \cdot \cos(\omega t)$

substitute into the ODE to obtain

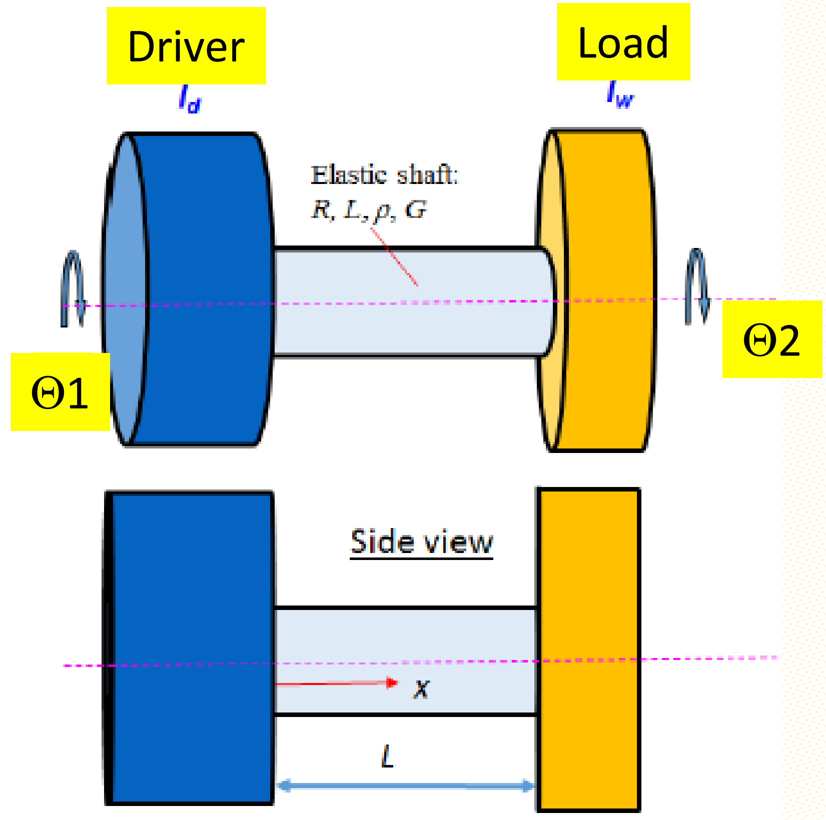
$$\begin{pmatrix} K_\theta - I_d \cdot \omega^2 & -K_\theta \\ -K_\theta & K_\theta - I_w \cdot \omega^2 \end{pmatrix} \cdot \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

for a nontrivial solution, the determinant of the algebraic equations must be zero,i.e

$$\Delta(\omega) = \left(K_\theta - I_d \cdot \omega^2\right) \cdot \left(K_\theta - I_w \cdot \omega^2\right) - K_\theta^{\,2} = 0$$

expanding the determinant one obtains:

$$K_\theta^{\,2} - K_\theta \cdot \left(I_w + I_d\right) \cdot \omega^2 + I_d \cdot I_w \cdot \omega^4 - K_\theta^{\,2} = 0$$



Graph not to scale

Natural freqs & mode shapes

or simplifying,

$$\left[I_d \cdot I_w \cdot \omega^2 - K_\theta \cdot (I_w + I_d)\right] \cdot \omega^2 = 0$$

corresponding to a rigid body mode $\psi_1 := \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Hence, the first root states $\omega_1 = 0$

i.e., both wheels move or turn with the same angle (no shaft twist)

The second root or eigenvalue follows from $I_d \cdot I_w \cdot {\omega_2}^2 - K_\theta \cdot (I_w + I_d) = 0$

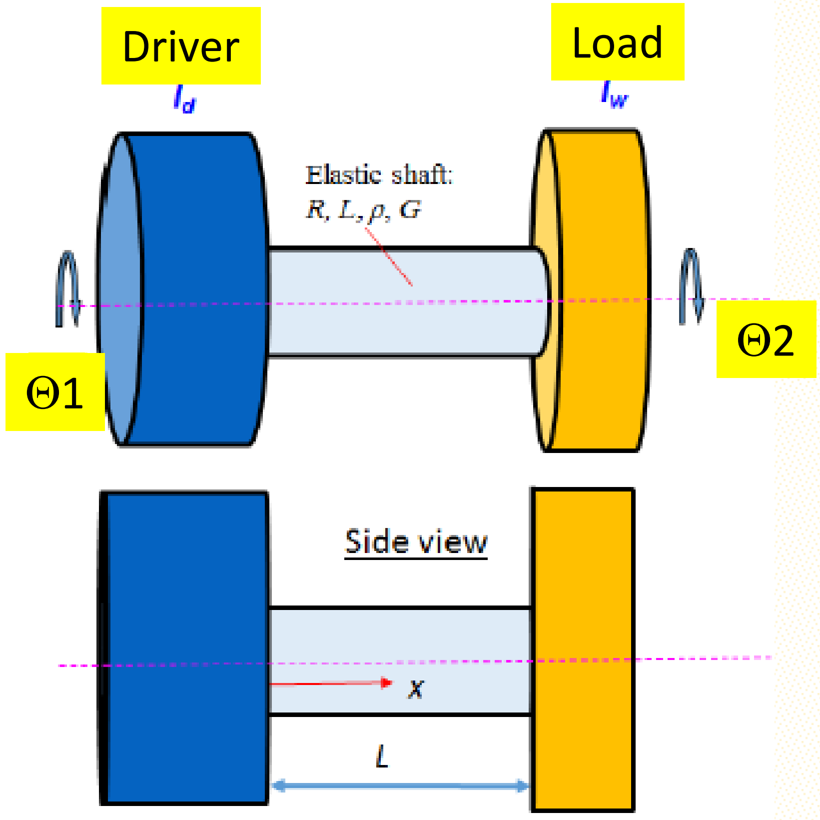
Define $I_{eq} := \left(\frac{1}{I_d} + \frac{1}{I_w}\right)^{-1}$, $= \frac{I_w + I_d}{I_d \cdot I_w}$

Hence, the elastic natural frequency is

$$\omega_n := \left(\frac{K_\theta}{I_{eq}}\right)^{0.5} = 679.366 \frac{1}{s}$$

substitute ω2 in the EOM to obtain the elastic mode eigenvector as $\psi_2 := \begin{pmatrix} 1 \\ \frac{-I_d}{I_w} \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

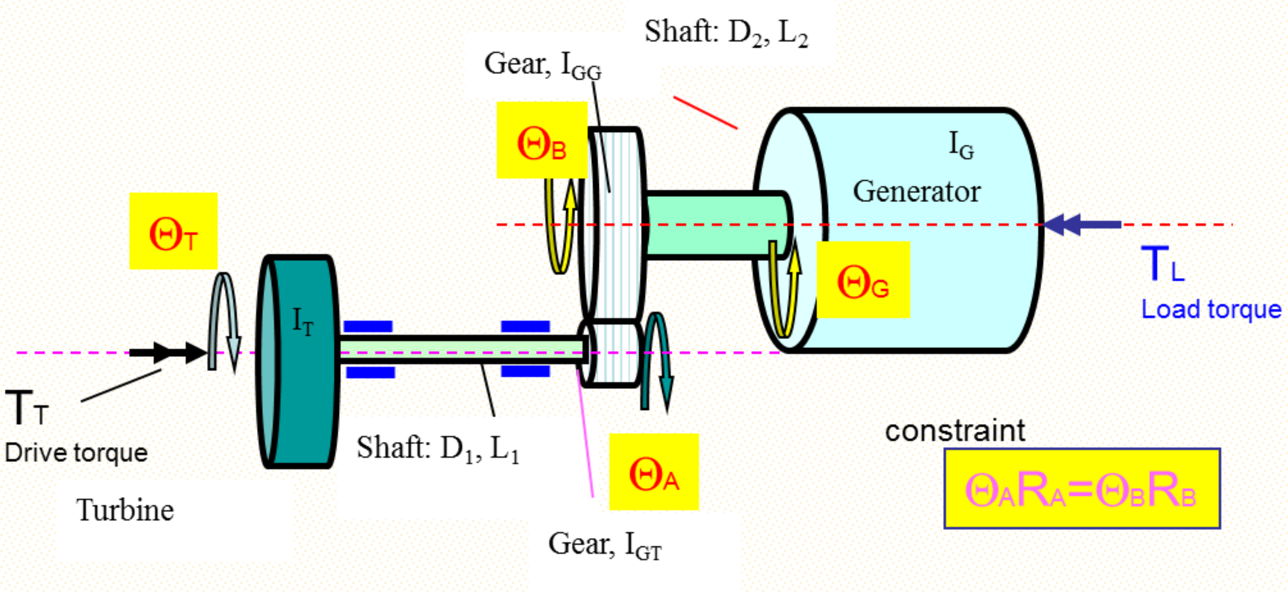
The w-wheel twists twice as much as the d-wheel (because it has less inertia)



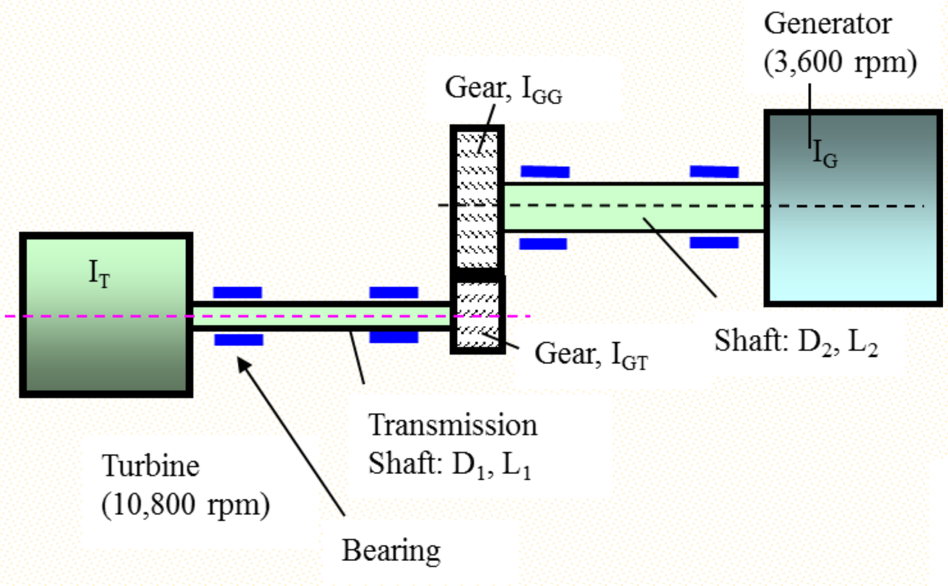
Graph not to scale

Torsional systeem with gears

Turbine (high speed) drives a generator (low speed) through a gear box

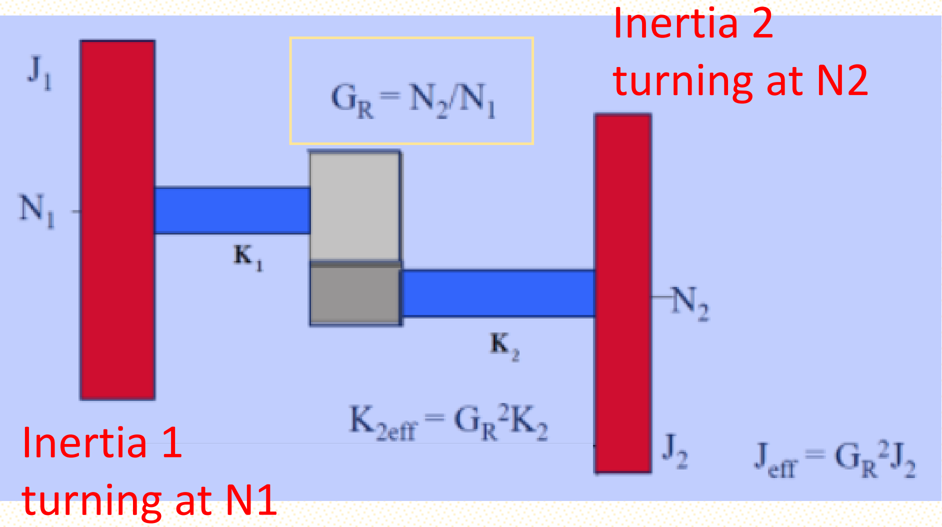


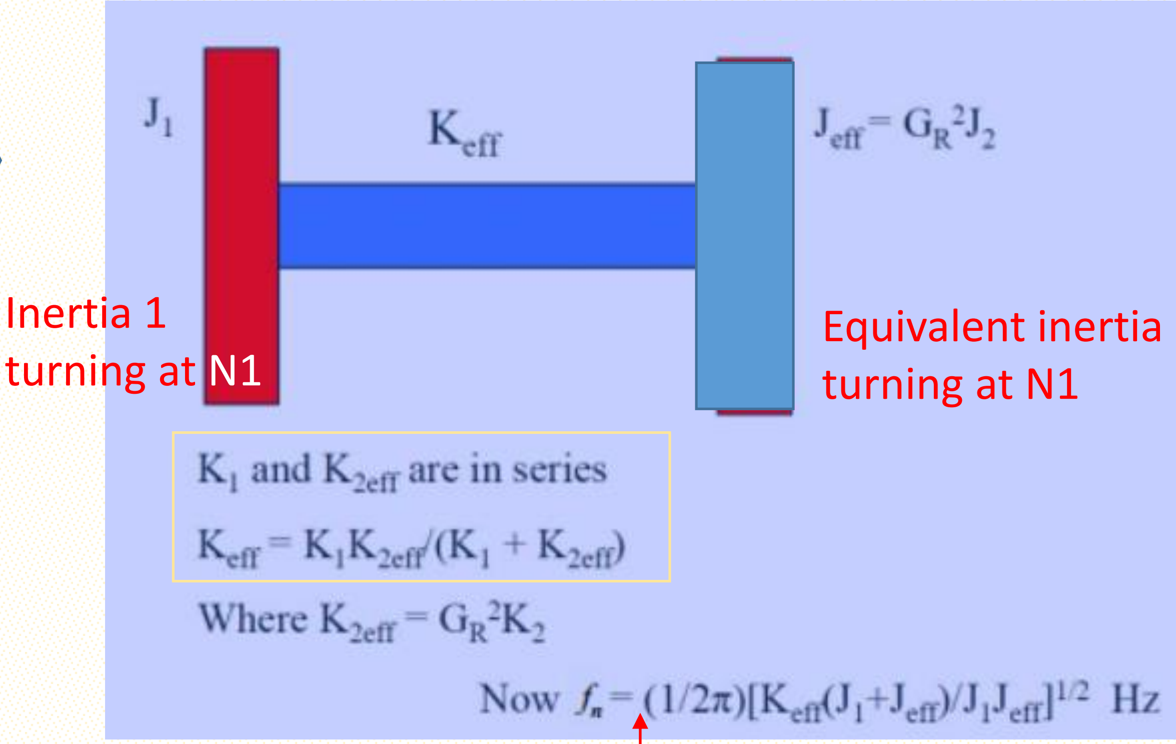
Isometric view with DOFs and top view (not to scale)



Torsional models with gears

Drive trains with gears can be reduced to an **equivalent system** (single shaft), provided that **J_1 and J_2 >> gear inertias.**





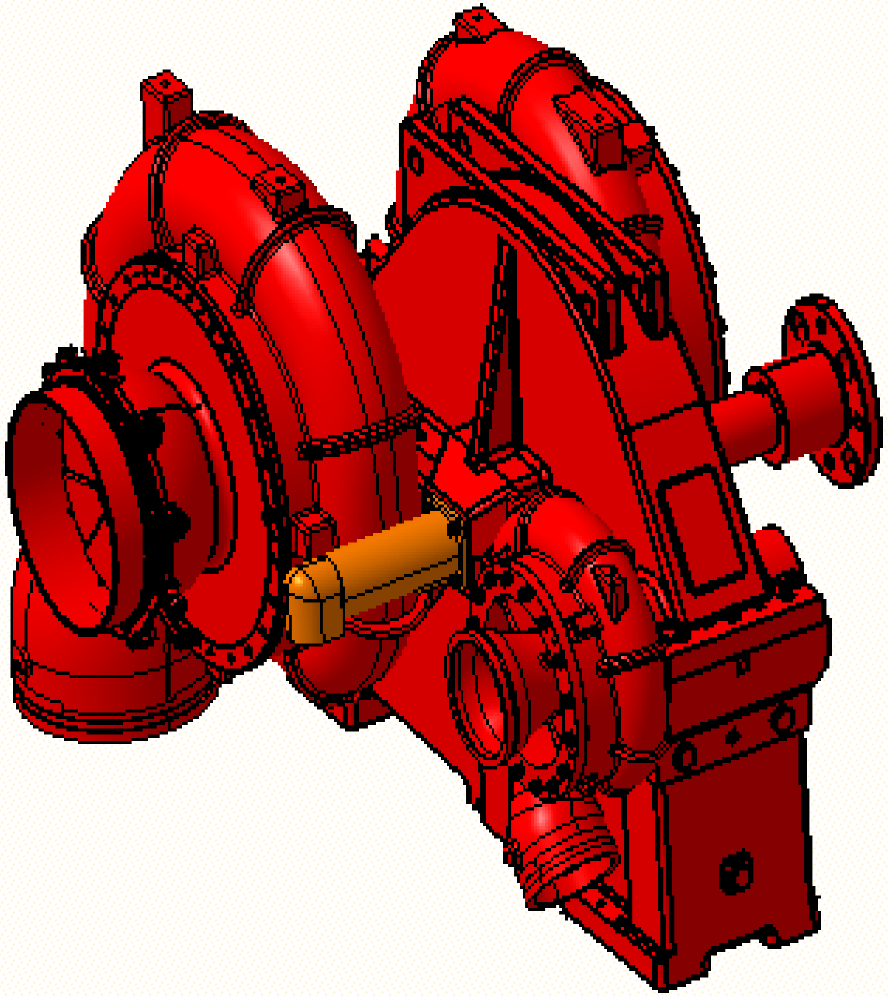
The gearbox amplifies the motion of the high speed shaft relative to the low speed shaft. G_R>1

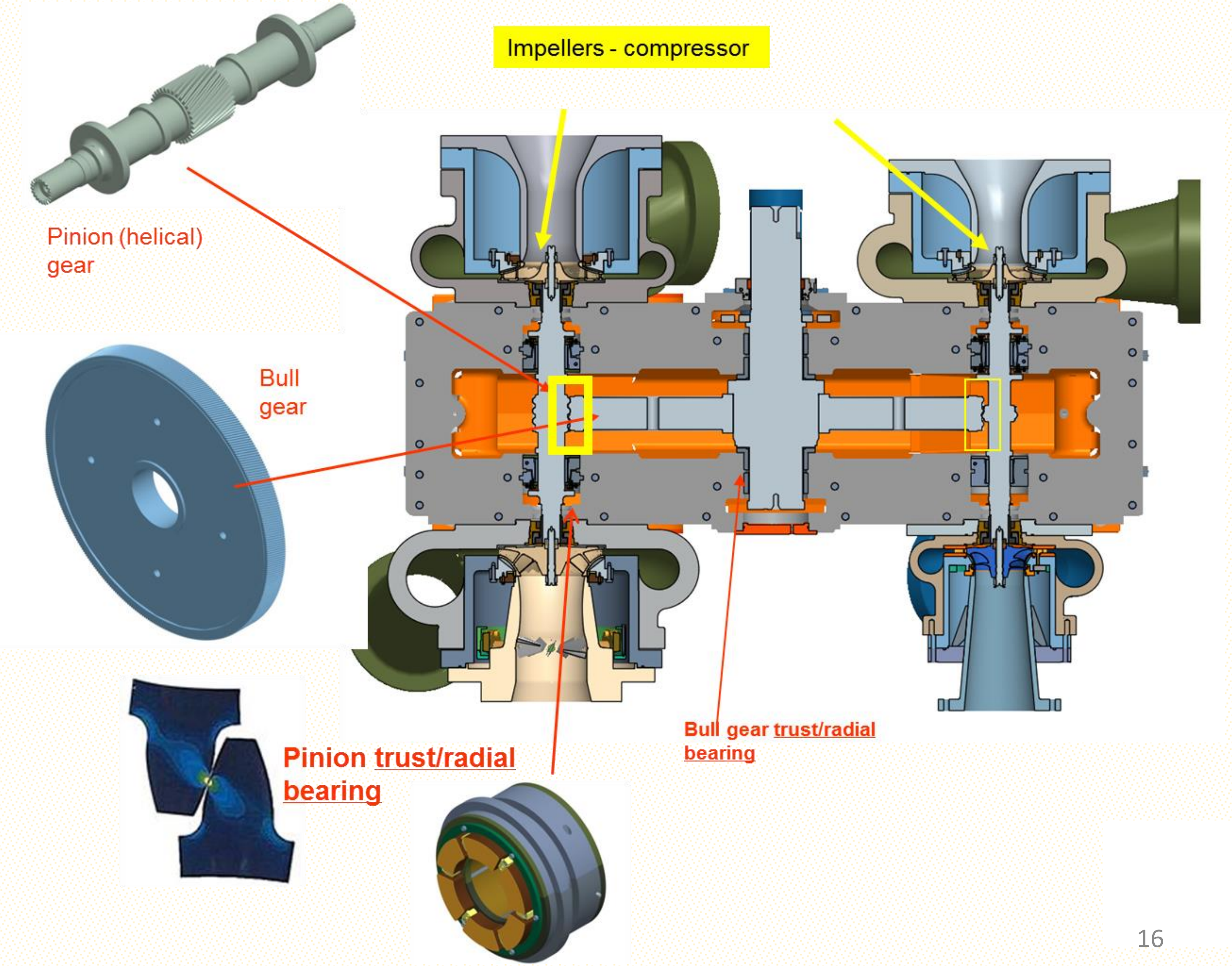
The easiest solution is to adjust the stiffness and inertia properties of one shaft to that of an equivalent shaft running at the same speed as the other shaft.

The adjusted parameters can be then used with the simple expression for f_n

Integrally geared compressor (IGC)

PICTURES courtesy of SAMSUNG Techwin



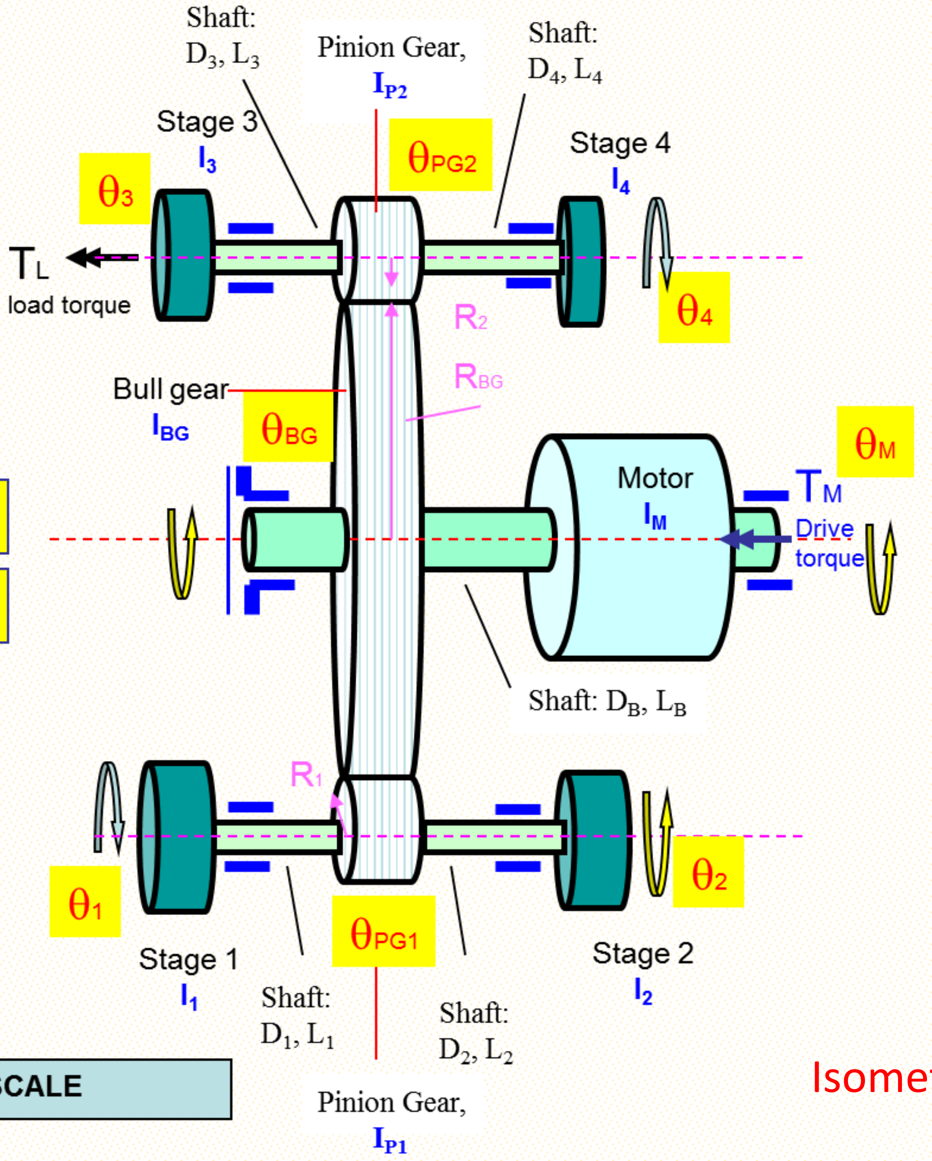


IGC Torsional model

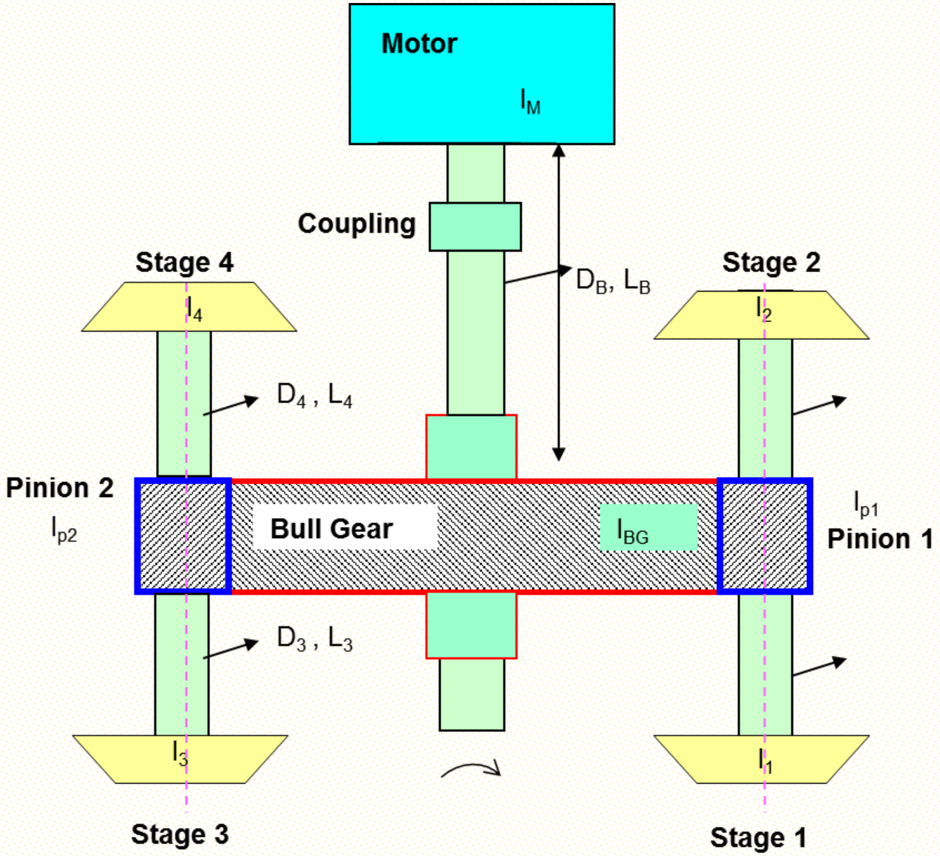
constraints

$\theta_{PG2}R_2=\theta_{BG}R_{BG}$

$\theta_{PG1}R_1=\theta_{BG}R_{BG}$

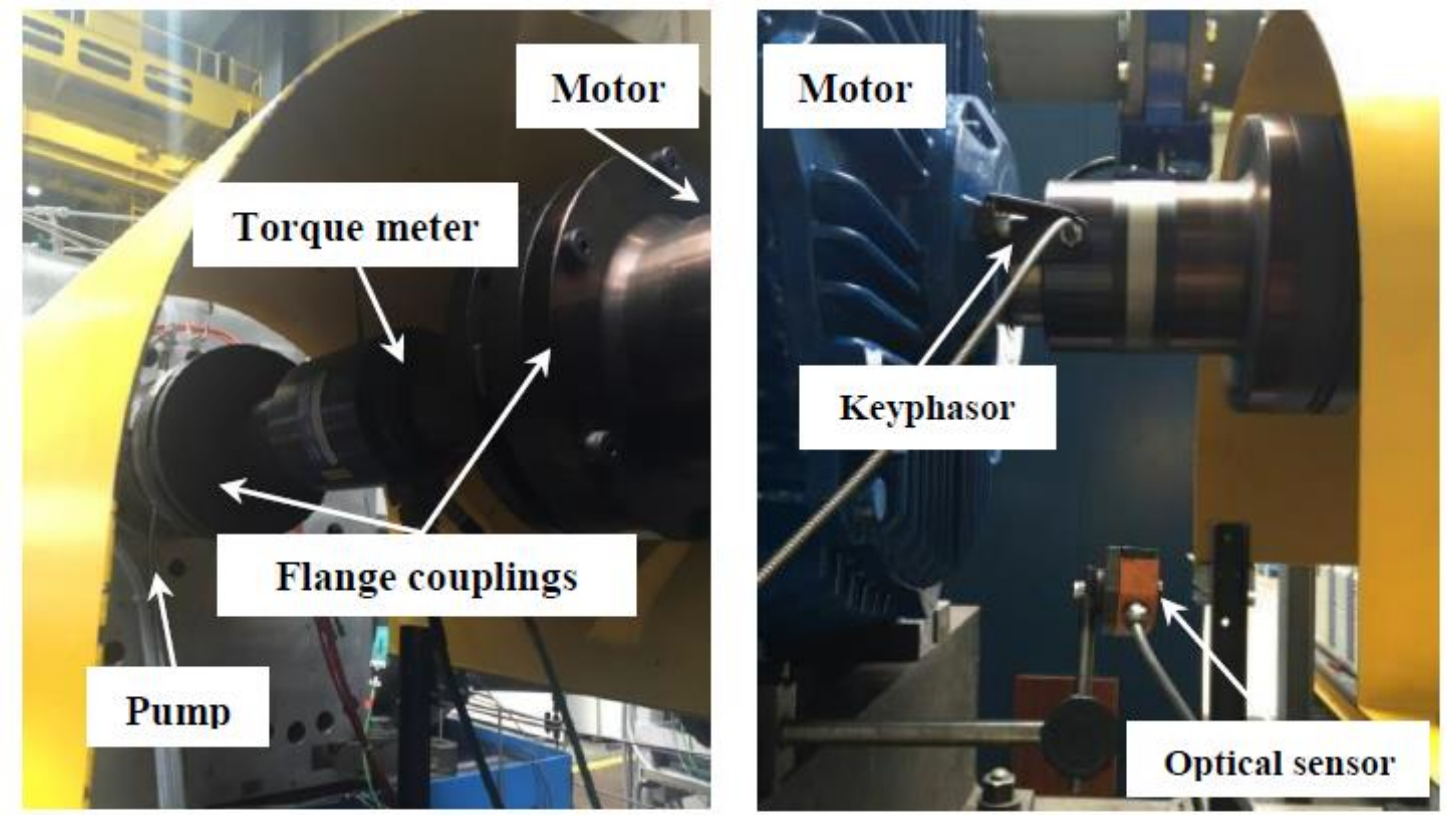


DRAWING NOT TO SCALE



Isometric view with DOFs and top view (not to scale)

Measurement of Torque and Angular Speed



Picture from 2019 Pump Lecture (M. Sciancalepore et al. Sulzer

Measurements of Torque and Ang Speed

Measurement of (drive) torque is most important in many applications as

Power = Torque x Angular speed → $P = T_o\ \Omega$

Angular speed Ω is easily recorded using a tachometer (mechanical or electromechanical or electronic → digital).

Nowadays most tachometers are rather inexpensive and use either infra-red light or diodes to detect the passage of dark/light regions – these sensors typically count pulses.

Other most advanced techniques include fiber optics and laser beams.

Measurement of Torque

Measurement of steady (drive) torque is customary in dynamometers used to record the power delivered to a machine (**$P = T_o\ \Omega$**). This power, of course, means a cost $$ to the end user (as in $x/kWh).

Many (static) torquemeters require the machine to be installed in (low) friction bearings or supports with the torque determined by multiplying a reaction force (F) x arm length.

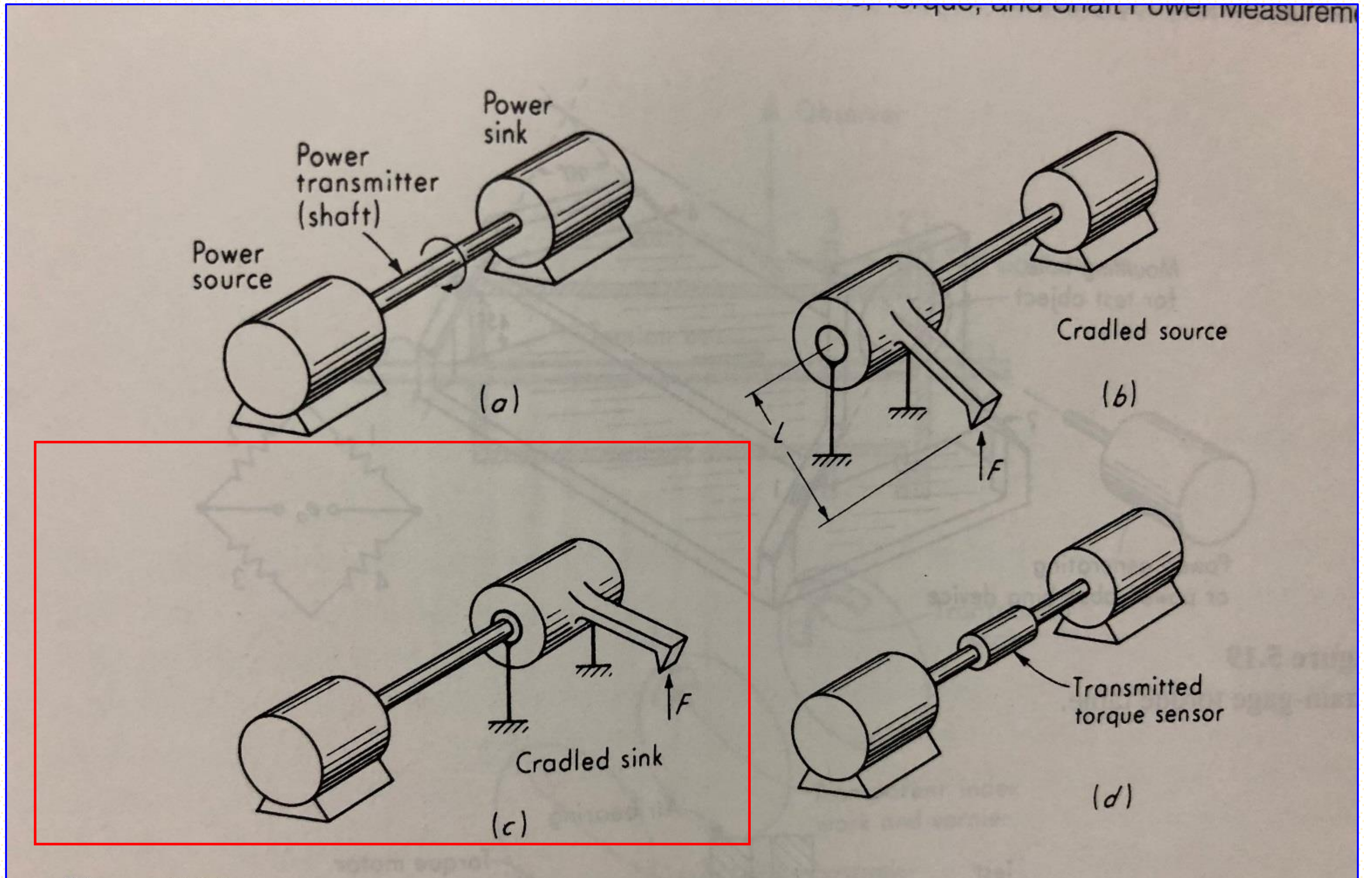


Figure 5.18
Torque measurement of rotating machines.

Figure from Doebelin, Measurement Systems, 5hth edition

A typical dynamometer

Power = Torque x Angular speed

→ $P = T_o \ \Omega$

Typical configuration used to test power and efficiency of internal combustion engines (ICEs).

The dyno provides a *load to* test the driver.

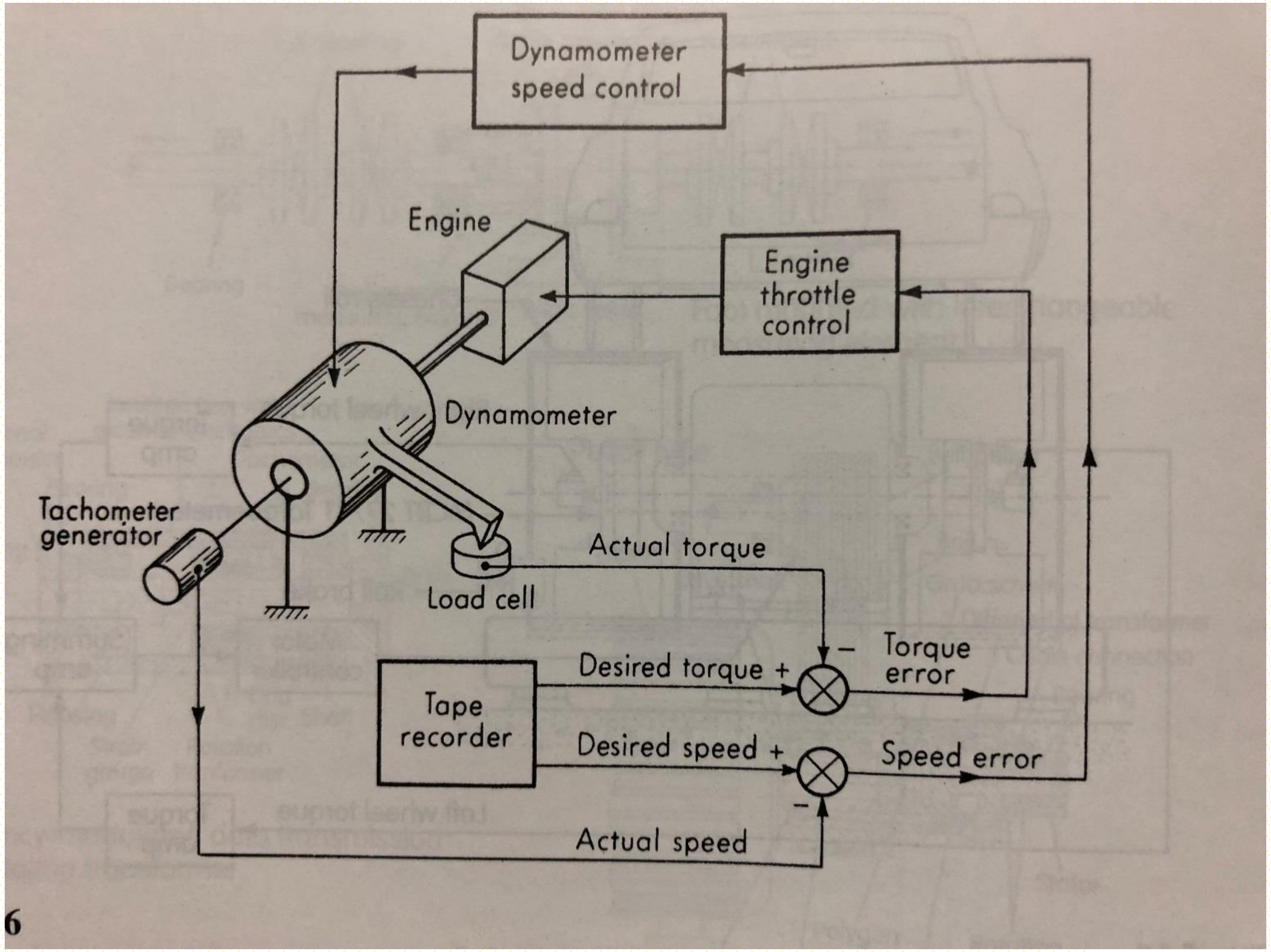
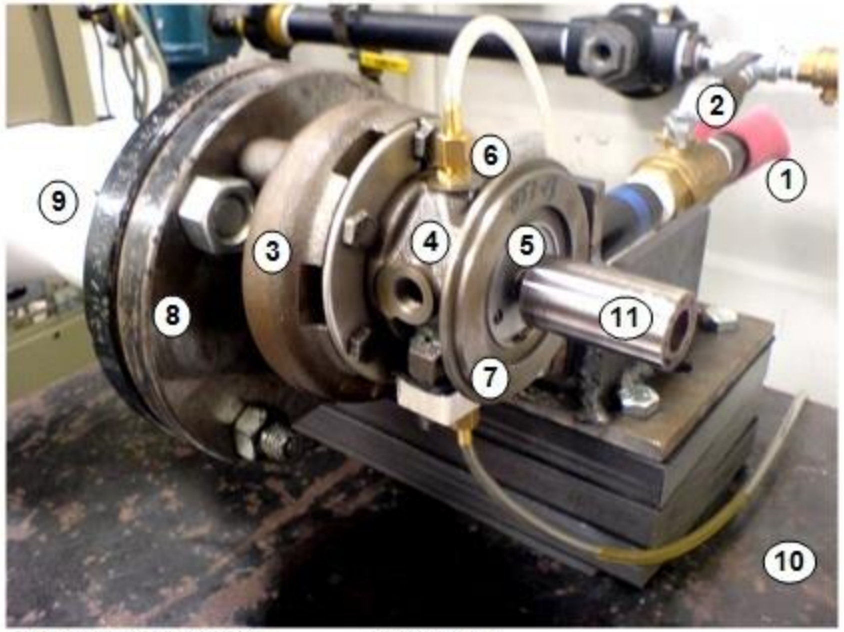
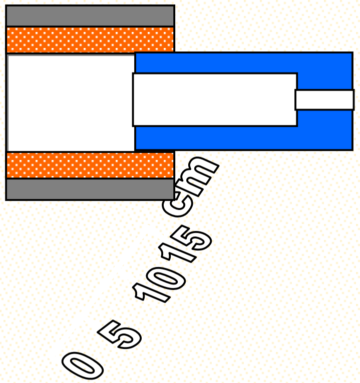


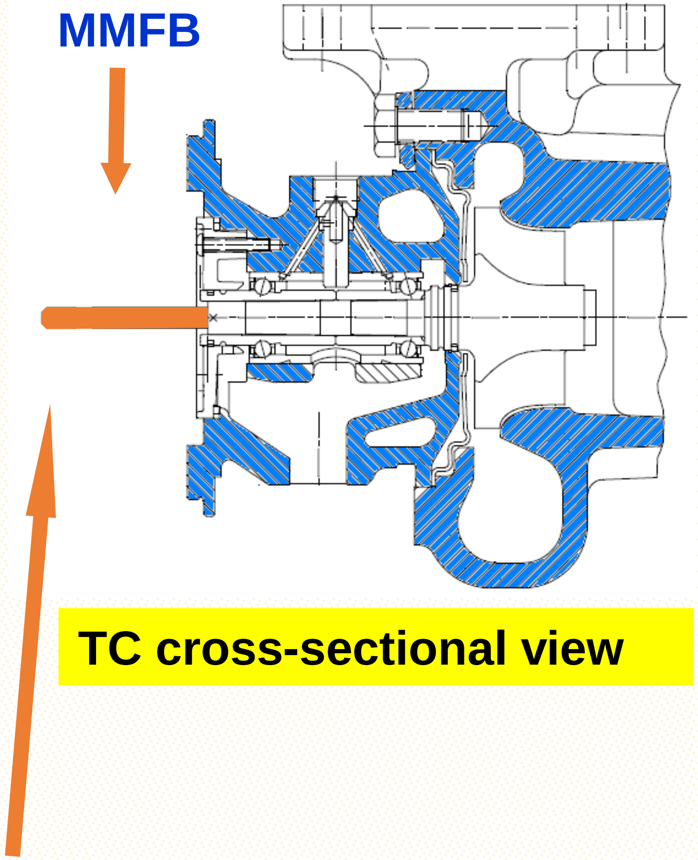
Figure from Doebelin, Measurement Systems, 5hth edition

A lab gadget for torque & lift off speed



1. 9.30 bar Air Supply
2. Throttle Valve
3. Turbine Housing
4. Center Housing
5. Stub Shaft
6. Oil Inlet
7. Oil Outlet
8. Turbine Outlet Safety structure
9. Turbine exhaust
10. ¾" Thick Steel Tabletop
11. Test journal (28mm outer diameter hollow shaft)





TC cross-sectional view

Journal press fitted on Shaft Stub

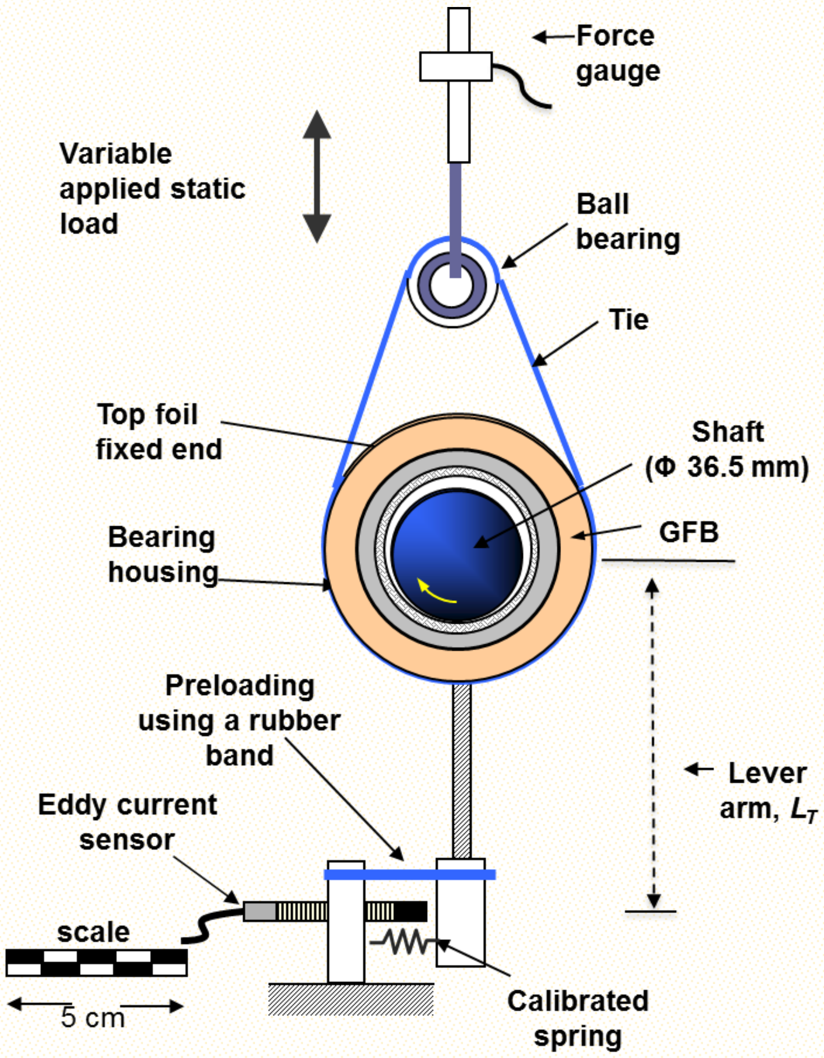
Max. operating speed: 75 krpm
Turbocharger driven rotor
Regulated air supply: 9.30bar

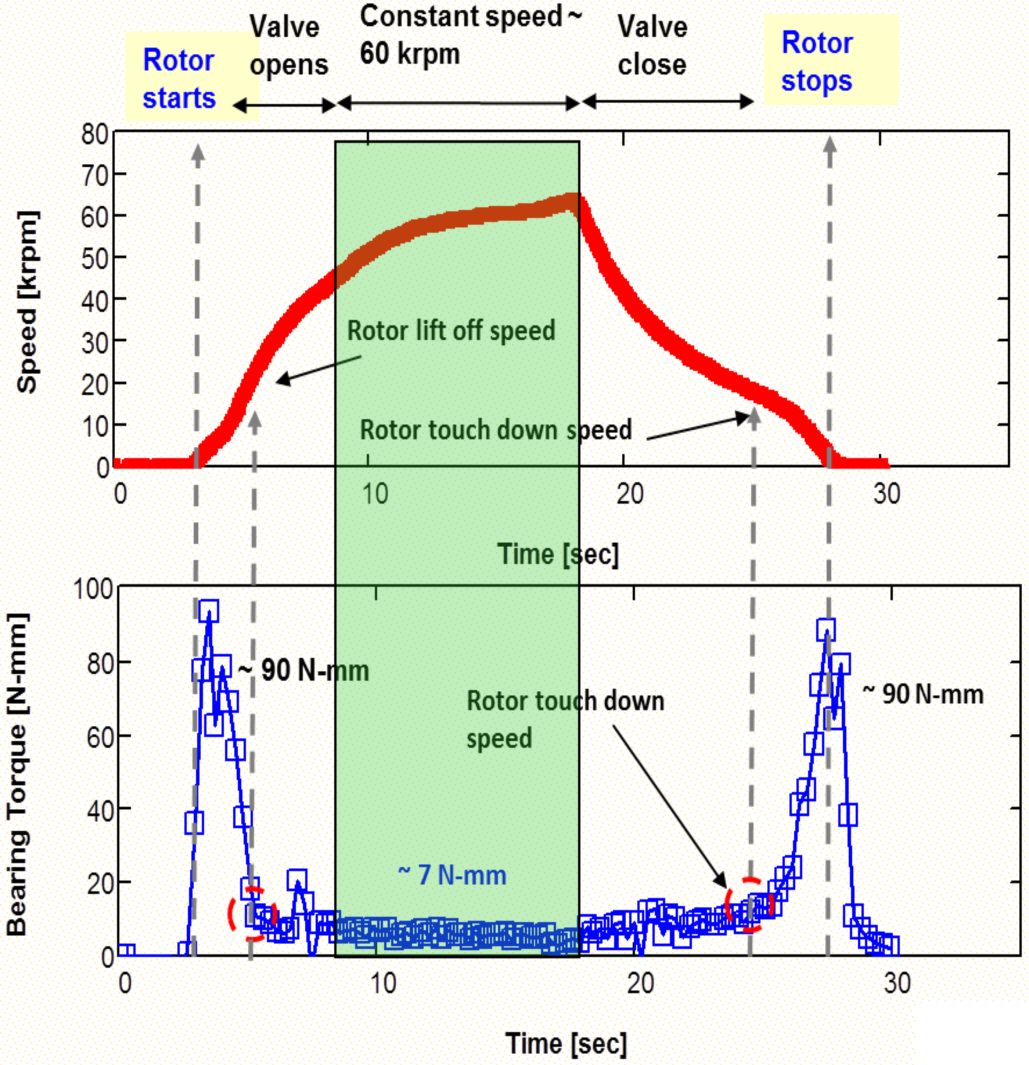
Journal: length 55 mm, 28 mm diameter , weight=0.22 kg

Twin ball bearing turbocharger
Model T25

Drag Torque = Force x Arm length = (K δ) x L

Accelerate TC to 60 krpm and decelerate to rest.

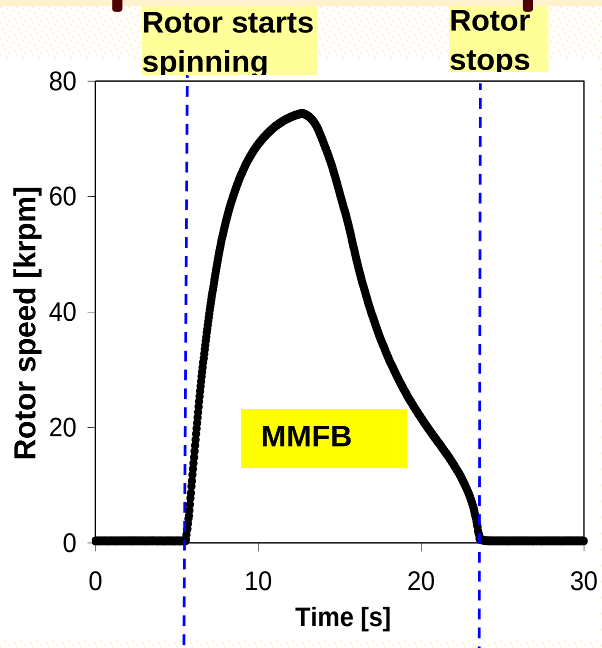




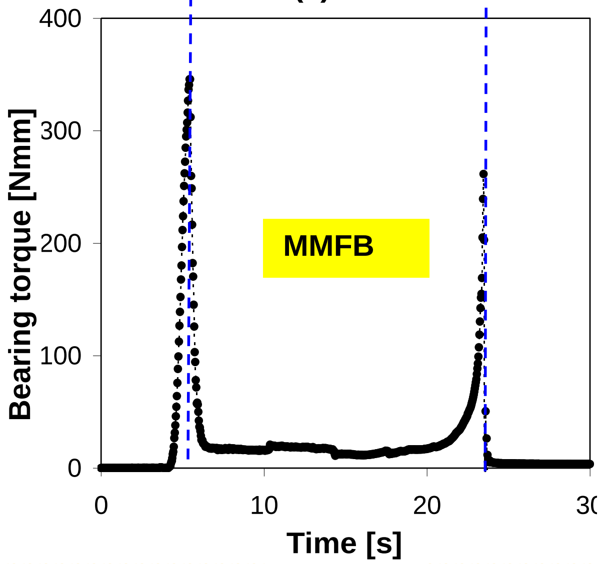
Lift off speed occurs at the lowest torque: airborne operation

Torque and speed vs time

Metal mesh foil bearing

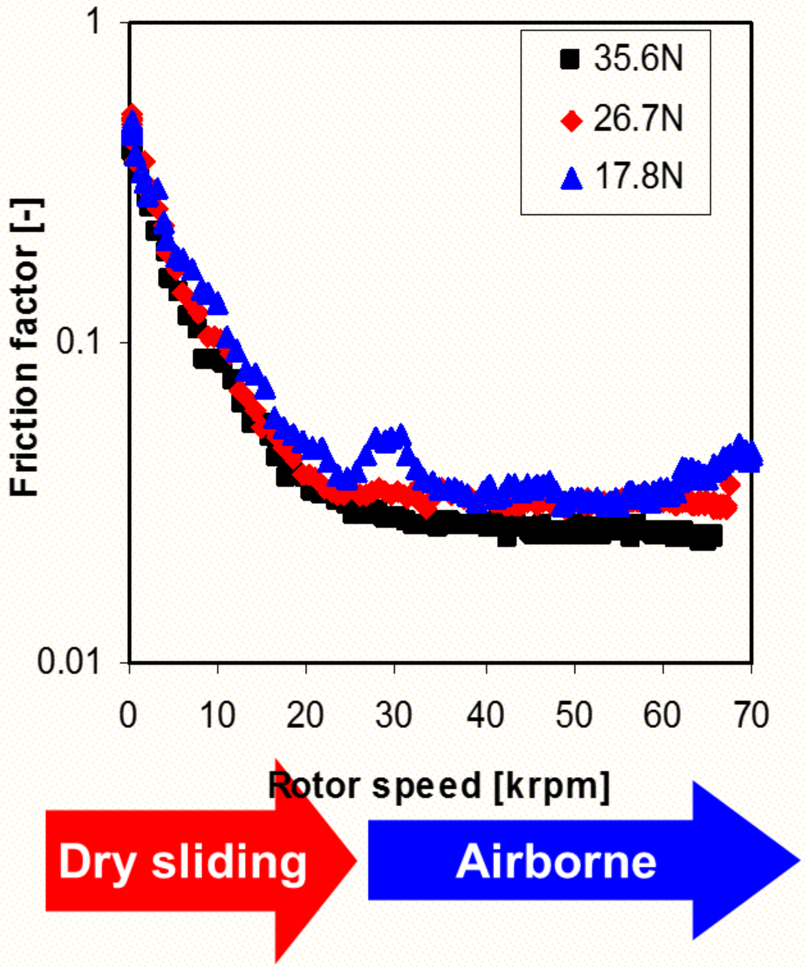


(a)

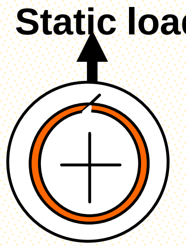


(b) MMFB torque

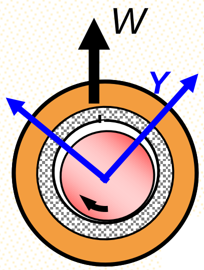
f **= (Torque/Radius)/(Static load)**



36 N load



Measurement to determine torque and rotor lift off speed from bearings → **friction coefficient**

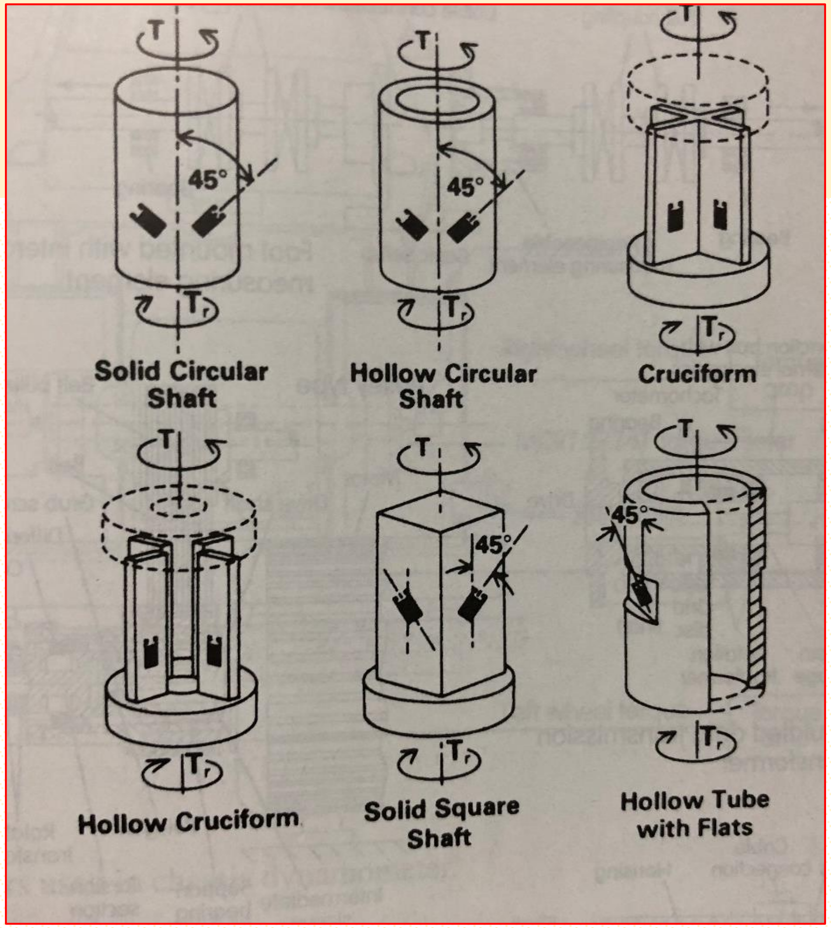


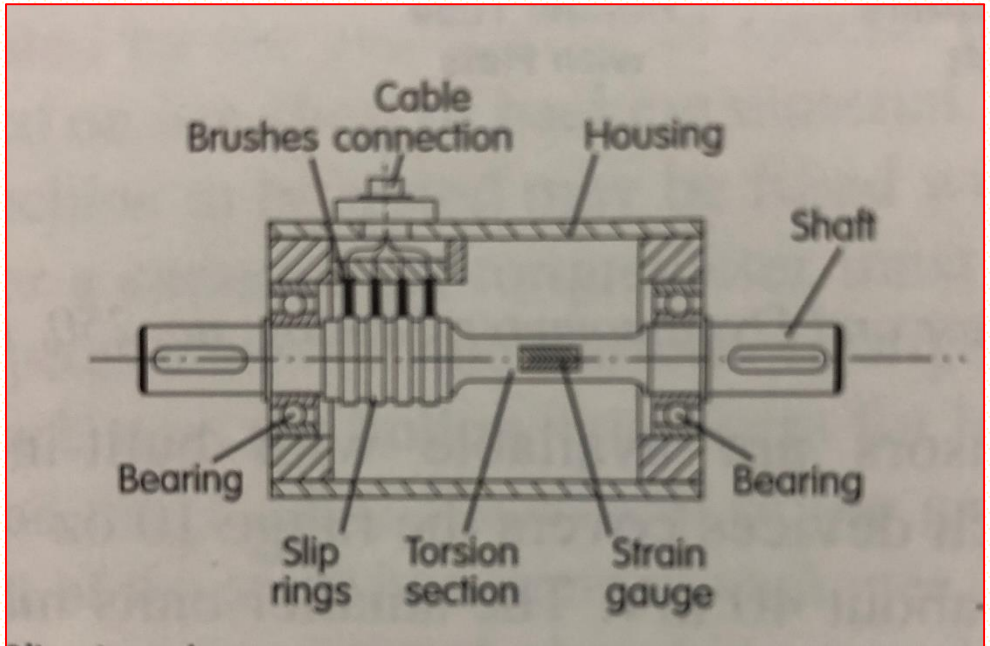
Torquemeter dynamic

Based on strain deformation of a transmission shaft

Install & calibrate strain gauges.
Route the signals out via slip rings.

(limited to low speeds < 12 krpm)





Figures from Doebelin, Measurement Systems, 5hth edition

A modern torquemeter flange

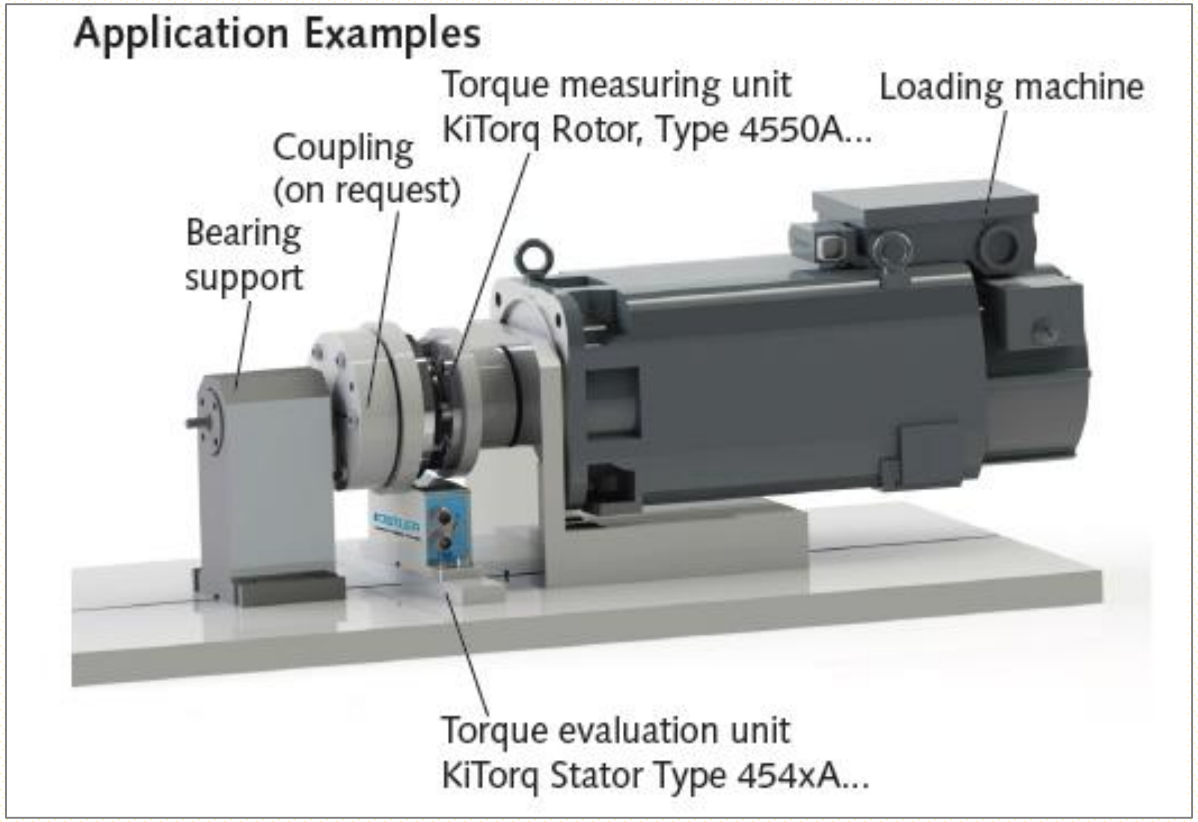
Figures from Kistler

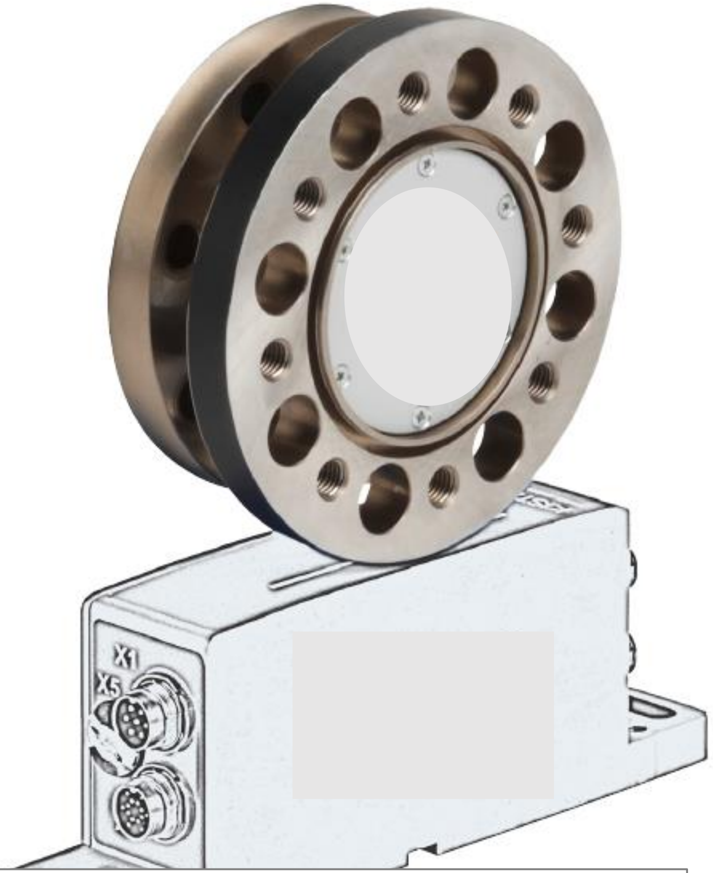
Strain gauge, no contact, low mass

static and dynamic torque (to 20 krpm)

wireless transmission of data (35 ksamples/s)

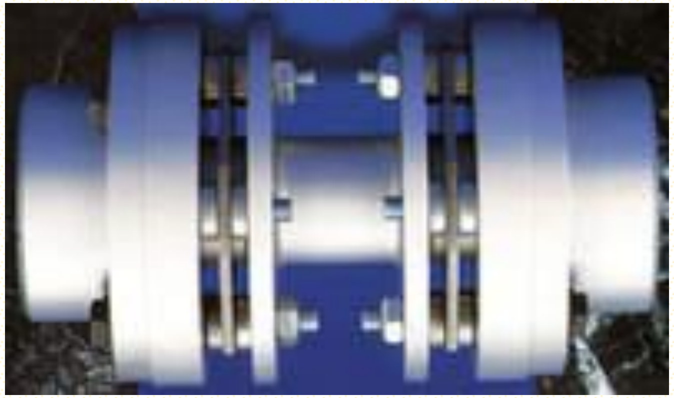
Costly $$



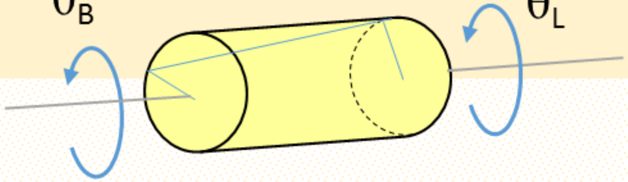


Mechanical Basic Data				
Type 4550A...			100...	200...
Rated torque	M_{nom}	N·m	100	200
Measuring range		N·m	±100	±200
Limiting torque[1)]	M_{op}	N·m	200	400
Rupture torque[1)]	M_{rupt}	N·m	>400	>800
Alternating torque	M_{dyn}	N·m	100	200
Nominal speed	n_{nom}	1/min	20 000	20 000
Torsional rigidity	C_T	kN·m/rad	231	349
Torsion angle at M_{nom}	φ	°	0,025	0,033
Max. bending torque[2) 3)]	M_B	N·m	30	50

Modeling Couplings



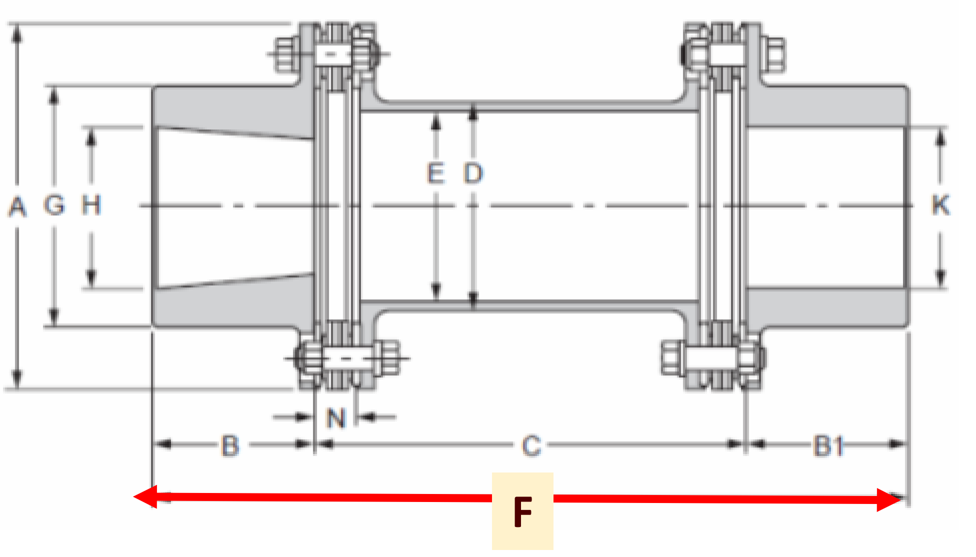
Modeling Flexible Couplings

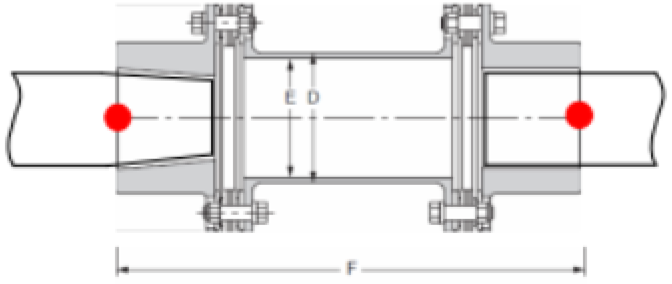


- Coupling torsional stiffness K_θ as provided by **coupling vendor** nearly always assumes **1/3** ***shaft penetration.***

- This means the stiffness includes everything in length F, including the stiffness of the shaft segment inside each hub.

- So attach the stiffness to each shaft at the red dots.





Stiffness and damping from e-motors

Models with Induction Motors

- k_M =electromagnetic stiffness to ground
- d_M =electromagnetic damping to ground

$$k_M = (\#\text{ stator poles})(T_B)\{x^2 / [1 + x^2]\}, \ldots \textbf{Eq'n (1)}, \text{ and}$$

$$d_M = k_M / (\omega^2 T_L) = k_M (T_L) / (x)^2 \ldots \textbf{Eq'n (2)},$$

where;

T_R = rated motor torque, [Nm]
T_B = breakdown torque. [Nm]
s_R = slip at rated load, [%],
Ω_s = supply frequency, [rad/s]

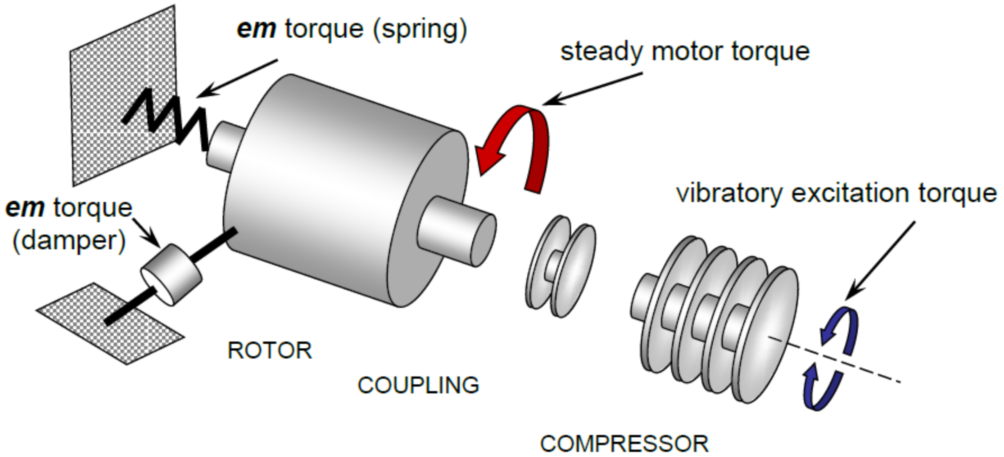
ω = torsional vibration freq., [rad/s]
T_L = electrical time constant, [s],
x = (ωT_L), dimensionless time.

The electrical time constant of the motor T_L can be estimated from:

$$T_L \approx (1/\Omega_s)[1/(2s_R)](T_R / T_B) \ldots \textbf{Eq'n (3)}.$$

[*] 2015 Estimates Of Electromagnetic Damping Across An Induction Motor Air Gap For Use In Torsional Vibration Analysis, Ed Hauptmann, Brian Howes, Bill Eckert, Gas Machinery Conference

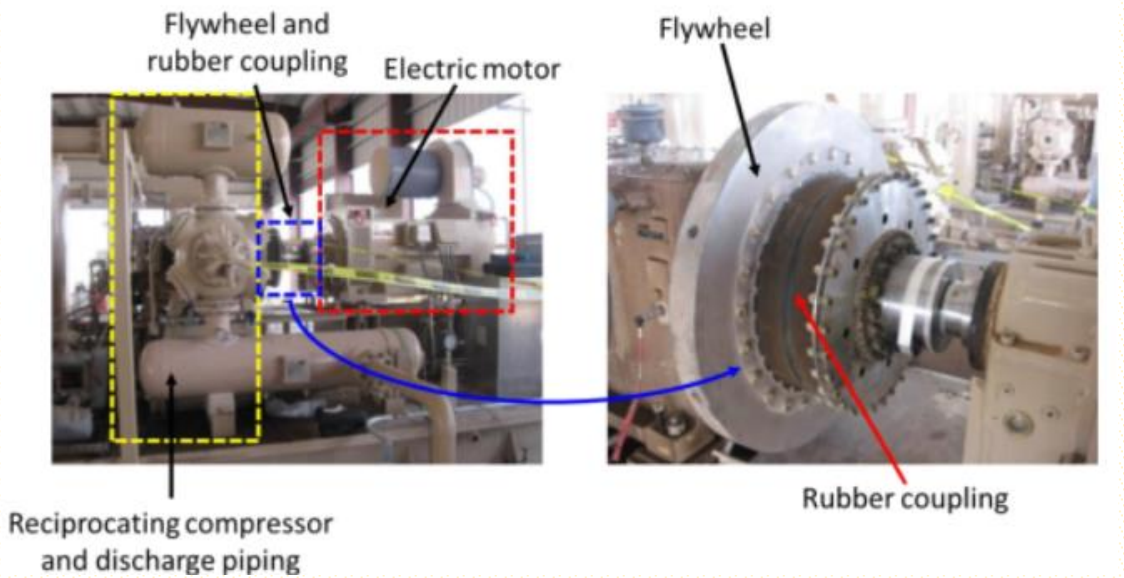
- ***electromagnetic stiffness and damping*** from induction motors.
- Expressions of stiffness and damping developed for reciprocating compressors but applicable to any induction motor.
- In turbomachinery applications, this effect is typically ignored. Situations where the em stiffness can significantly affect the 1st torsional mode are trains using very soft elastomeric couplings (see homework)

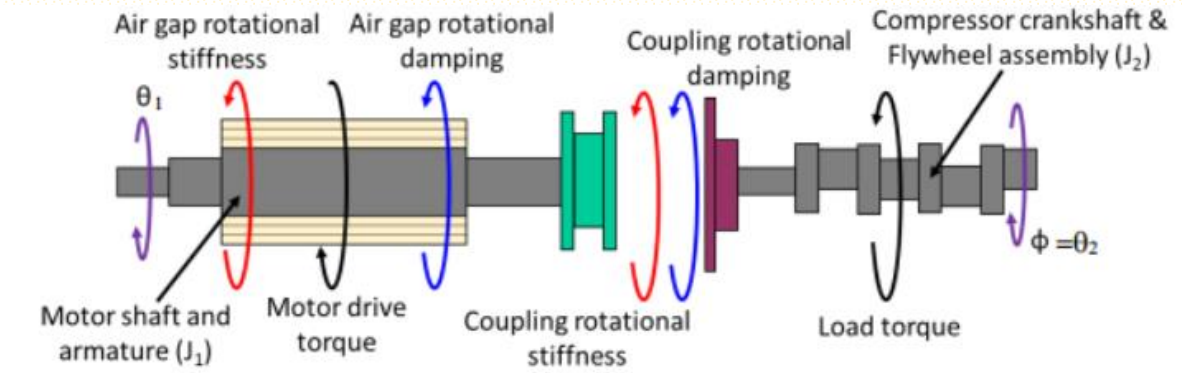


EM torsional example

[*] Feese, T., and Kokot, A., 2016, "Electromagnetic Effects on the Torsional Natural Frequencies of an Induction Motor Driven Reciprocating Compressor with a Soft Coupling," Proc. 45th Turbomachinery & 32nd Pump Symposia, Houston, TX., September, pp. 1-22

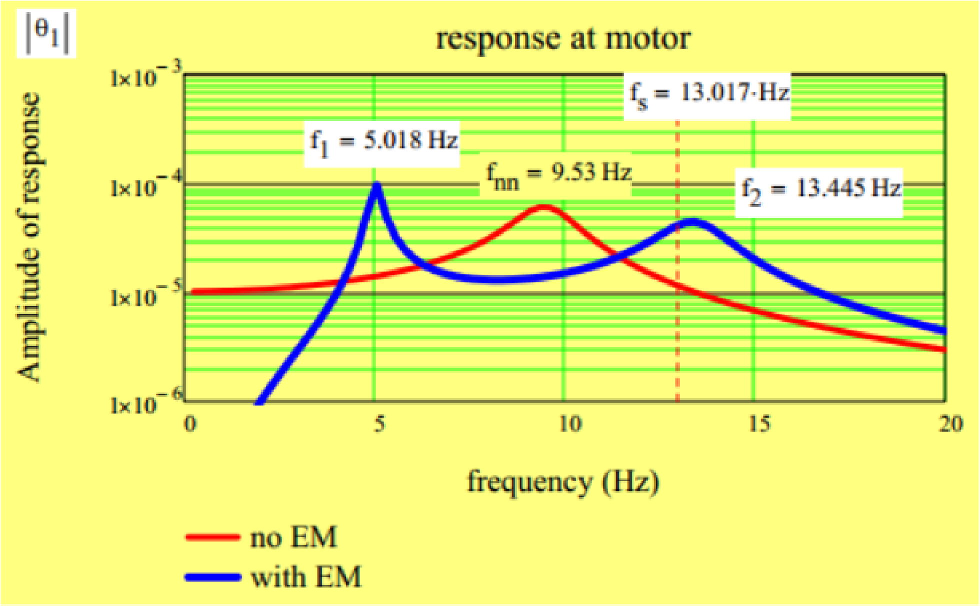
Photographs of a motor driven reciprocating compressor.





Torsional model.

- Compressors exhibited excessive torque fluctuations.
- Torsional analysis at design stage omitted electromagnetic (EM) torsional stiffness and damping.
- Analysis including EM force coefficients revealed system was operating at a torsional natural frequency



Are torsional systems stable?

Most torsional systems, even having a small (torsional) damping are stable.
However, there are important exemptions

Self-Excited Torsional Vibration

Unstable Self-Excitation of Torsional Vibration in AC Induction Motor Driven Rotational Systems

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The most common NEMA Design Classes of AC induction motors have speed-torque characteristics that can give rise to unstable self-excitation of torsional vibration in rotational systems during start-up. A torsional vibration computational model for start-up transients has been developed as a design tool for induction motor applications. Torsional instability can occur at speeds in the positive sloping segment of the motor's speed-torque curve and is particularly acute when the mass moment of inertia of the load device is more than two times the mass moment of inertia of the motor rotor. The computational model is compared with an exact solution method and with a laboratory test of a motor-driven inertial load. Applications of the computational model to electric submersible pump (ESP) design cases are discussed.

Positive slope of motor torque curve produces negative damping (see next slide)

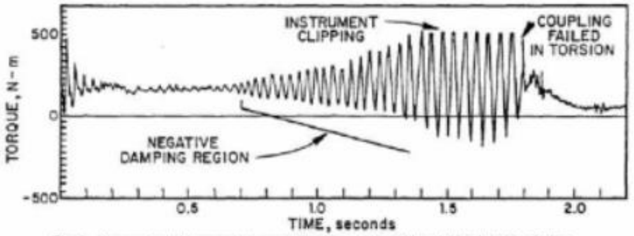


Fig. 3 Measured shaft torque between induction motor and inertial load during start-up

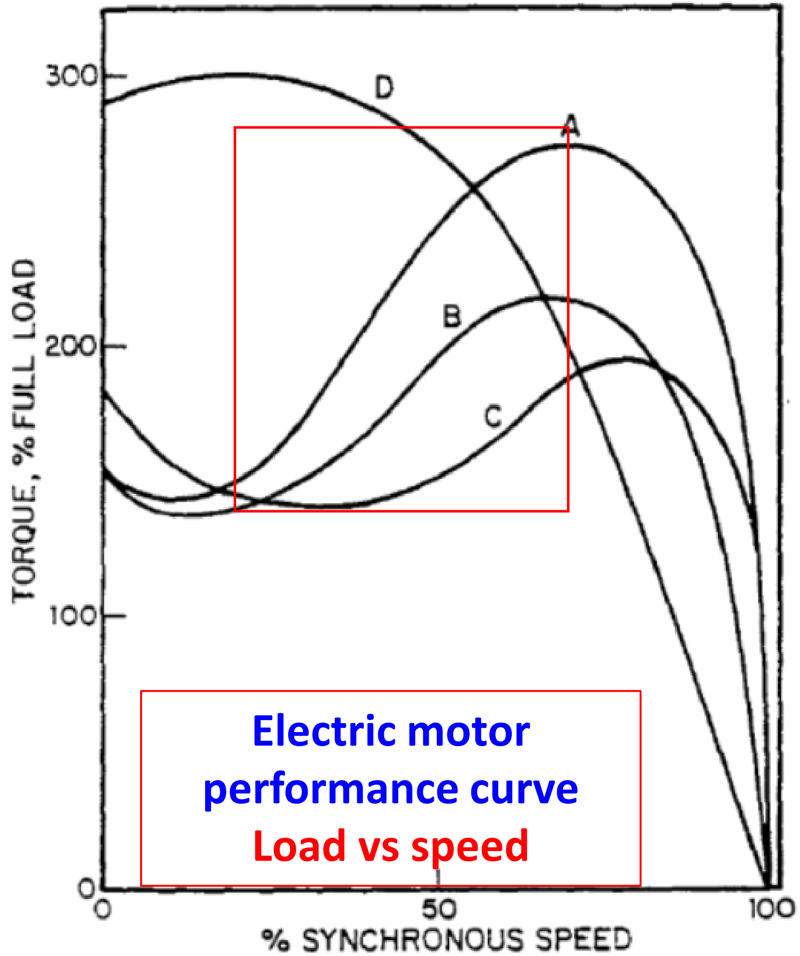


Fig. 1 Representative speed-torque curves for electric AC induction motors in NEMA design classifications A, B, C, and D

- **Positive slope of motor torque produces negative damping.**
- Class A motors are the most efficient, but also have the longest dwell time with a positive torque-speed slope, which produces negative damping.

Self-Excited Torsional Vibration

A one-degree-of-freedom system for which system instability is possible is one in which the excitation force is proportional to velocity as described in Eq. (1).

$$m\ddot{x} + c\dot{x} + kx = F(\dot{x}) = B\dot{x} \quad (1)$$

In Eq. (1), x, $\dot{x}$, and $\ddot{x}$, are the displacement, velocity, and acceleration of the system, m is the system mass, c is the viscous damping coefficient, and k is the stiffness coefficient. $F(\dot{x})$ in Eq. (1) is the forcing function and is equal to the coefficient, B, times the system velocity. By rearranging Eq. (1) into the form

$$m\ddot{x} + (c - B)\dot{x} + kx = 0 \quad (2)$$

one can recognize the possibility for negative damping, i.e., when B exceeds c. When B is greater than c in Eq. (2), the motion of the system tends to increase the energy of the system, and the amplitude of motion increases in unstable self-excitation.

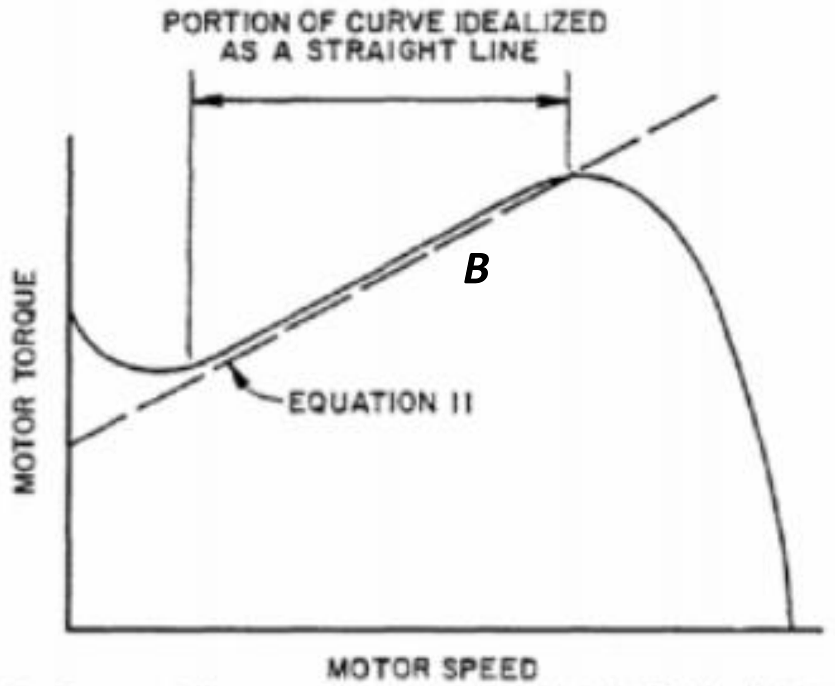


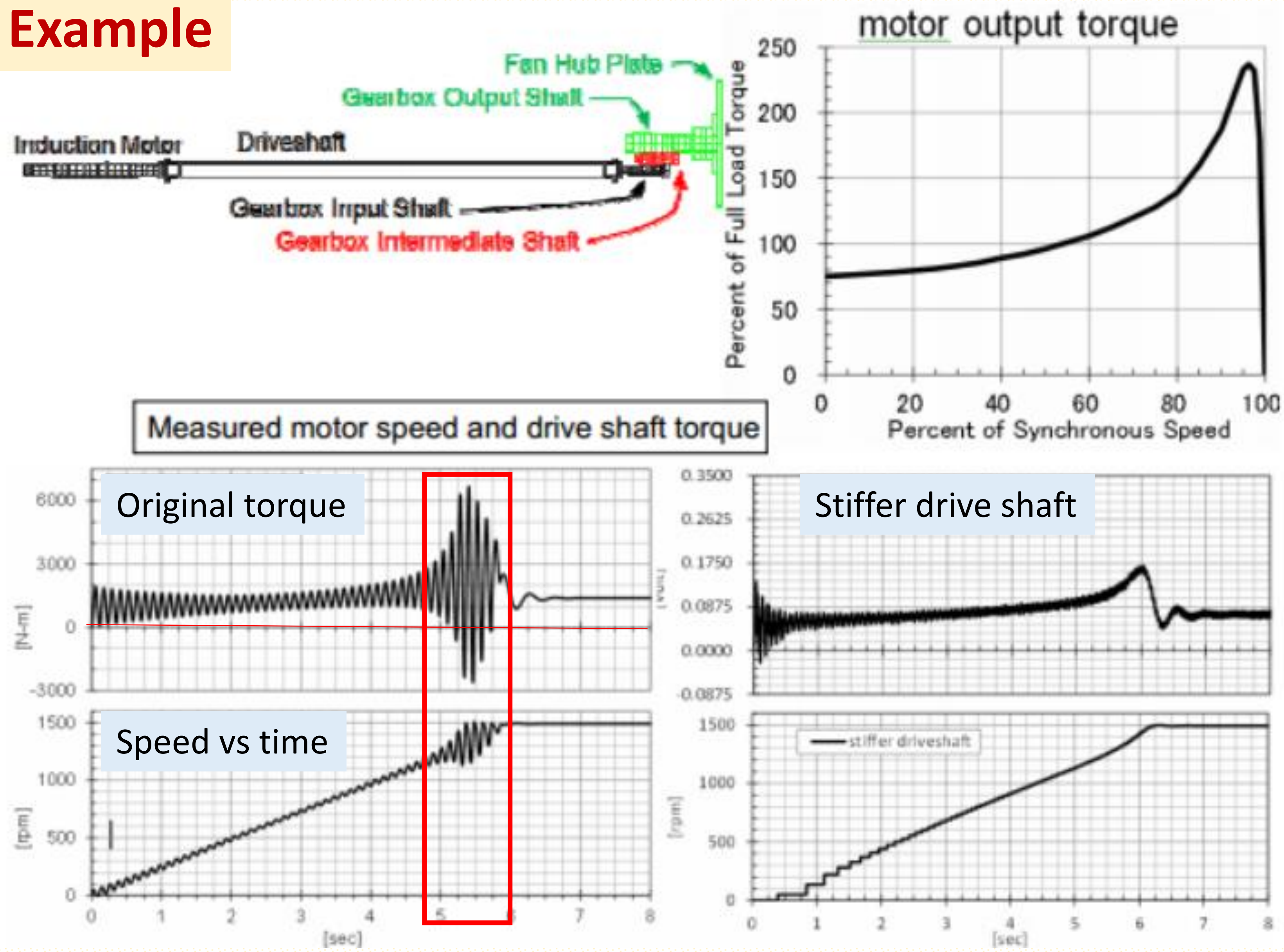
Fig. 2 Representative speed-torque curve for electric AC induction motors used in ESP systems

The analogous torsional motion equation is:

$$I_P\ddot{\theta} + (C - B)\dot{\theta} + K_T\theta = 0$$

where B is the slope of the linearized drive torque. The system is unstable if *B* > *C*.

Example



2016 TPS, "Torsional Instability of Cooling Tower Fan During Induction Motor Startup", Akira Adachi & Brian Murphy

- 220 kW cooling tower draft fan in a petrochemical plant.
- Induction motor (1490 rpm) and a two stage reduction gearbox.
- The unit is started direct online with 50 Hz power (i.e. no VFD)
- 14 of 16 units experienced significant gear damage during plant commissioning.
- Stiffer shaft increased nat. frequency from 8.4 Hz to 14 Hz

Synchronous Electric Motor Drive Trains

It is becoming increasingly attractive (due to energy considerations) to drive centrifugal or axial compressors and blowers with large synchronous electric motors. The shaft speed step-up is usually accomplished with a parallel-shaft helical gear set enclosed in a gearbox. Fig. 2.8 shows such a train.

Synchronous motors of the salient-pole type are started as induction motors, with the field short-circuited or discharged through a resistor. This produces a pulsating torque, during start-up, with a frequency which varies from twice line frequency initially down to zero at synchronous speed. Any natural frequencies of torsional vibration that lie within this range will be excited during the start-up. The magnitude of the driving torque pulsation varies with speed and varies from motor to motor, but can be quite large, with a peak-to-peak amplitude often greater than the average torque.

Mruk et al. [16] have measured the pulsations and found them to be even larger (by a factor of 5) for an abnormal shaft with malfunctioning exciter circuits. Under the latter condition the predominant frequency of the pulsations during start-up was equal to the motor slip frequency (one-half its usual value under normal conditions).

When this type of excitation becomes resonant with a natural frequency, even if for only a few seconds, it is not unusual for the gears in the train to clatter or even break as a result of tooth separation and impact. shaft failure due to this phenomenon, and Brown [18] has shown a compressor shaft failure.

from Vance, *Rotordynamics of Turbomachinery*

Rotordynamics of Turbomachinery, John Wiley & Sons, 1988, pp. 55.

- During start up, a synchronous motor produces pulsating torque that may excite torsional natural frequencies.

- API 546 covers synchronous motors over 0.5 MW. **A torsional analysis is required if the motor drives a reciprocating machine.** Otherwise it is up to the purchaser to specify whether or not to require a torsional analysis, and that includes the transient startup analysis.

Tasks in a Torsional Vibration Analysis

- **Determine critical speeds (excite a natural frequency) and mode shapes**
  - Basic eigenvalue calculation
  - Torsional interference diagram (Campbell diagram)
- **Predict shaft torque response due to generic shaft orders like 1X and 2X and where the magnitude of excitation is a % of nominal torque** (e.g. ½% or 1%)
- **Predict shaft torque response to transients**
  - numerical integration (time marching)
  - Machine train start up with synchronous motors
  - Electrical transients (starts, faults, etc.)
  - Perturbations due to harmonic distortion in Variable Frequency Drive Motors
- **Predict steady state shaft torque response in reciprocating machines**
  - Requires analyzing a multitude of torque harmonics throughout the operating speed range
  - Responses to many individual harmonics must be superposed

Pump: Torsional Damping Coefficient

- Pump power is 550 hp at 1800 rpm, and power varies with the cube of shaft speed. So torque as a function of speed is

$$T = \frac{P}{\omega} = \frac{c\omega^3}{\omega} = c\omega^2$$

$$c = \frac{550 * 6600\, in - lbf / s}{(\frac{\pi}{30} 1800 \frac{rad}{s})^3} = 0.542\, in - lbf - s^2$$

$$C = \frac{dT}{d\omega} = 2c\omega = 2c(\frac{\pi}{30} 1725 \frac{rad}{s}) = 195.8\, in - lbf - s$$

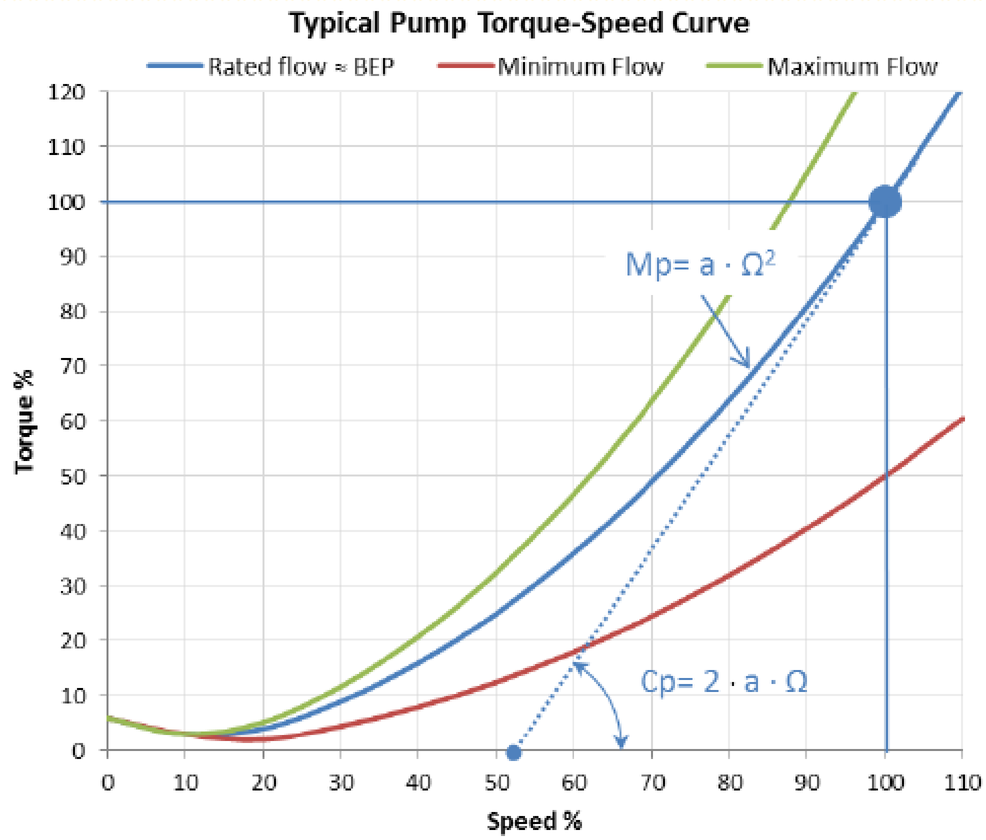


Figure 22: Pump impeller quasi-static damping from the torque-speed cur

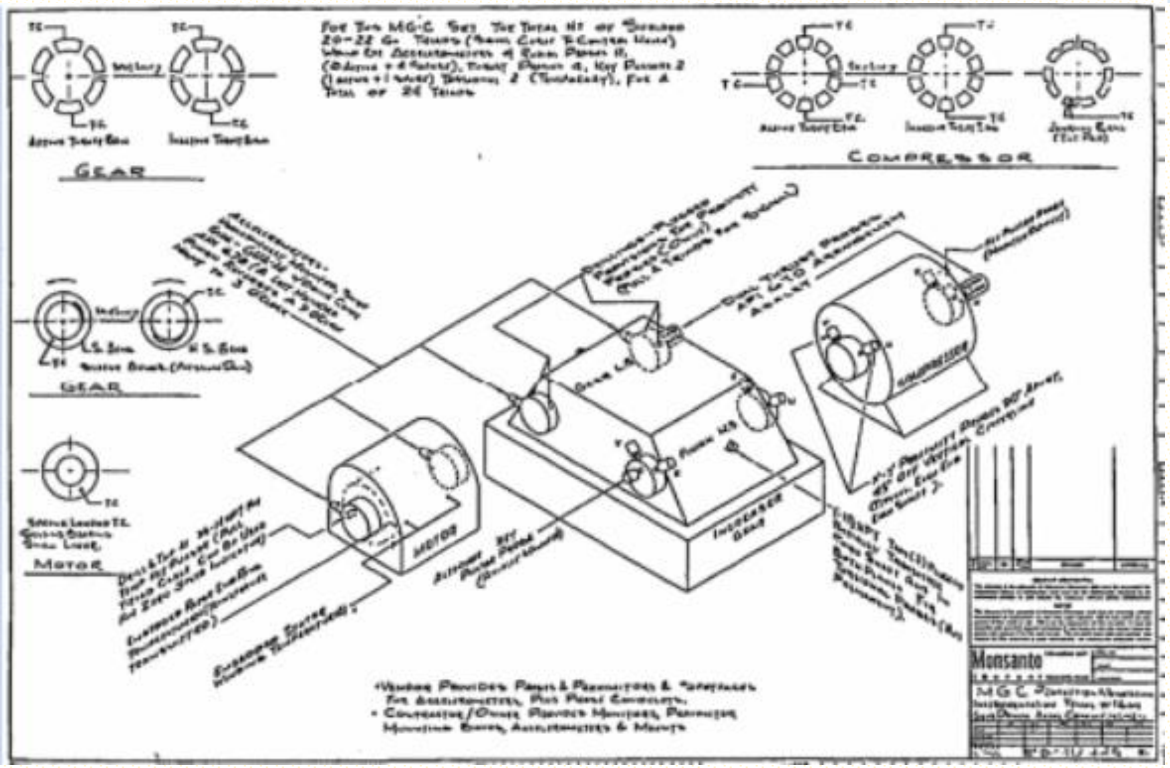
- *C* = equivalent viscous damping constant acting between the pump rotor and ground
  - **195.8 in-lbf-s damping, units = torque per rad/s**

When in doubt, In a torsional analysis assume

(TYP) 1% critical damping

Example - Synchronous Electric Motor Drive Train

Jackson and Leader, 1983 Design, Testing and Commissioning of a Synchronous Motor-Gear-Axial Compressor, Proc 12^{th} Turbomachinery Symposium.



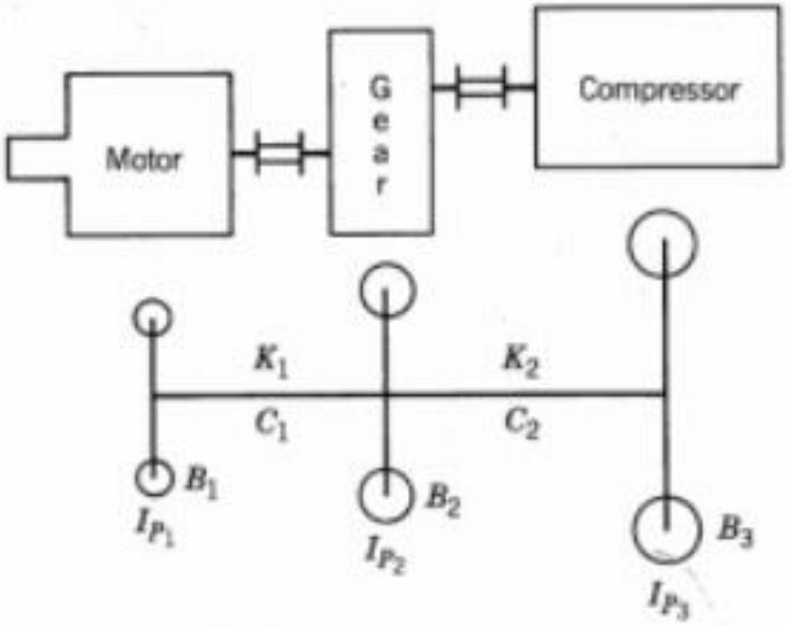


Figure 3.12. Industrial drivetrain with three-inertia torsional model.

$I_{P_1} = 4192$ in-lb-sec^2, $I_{P_2} = 4907$ in-lb-sec^2, $I_{P_3} = 10{,}322$ in-lb-sec^2

$K_1 = 73.59 \times 10^6$ in-lb/rad, $K_2 = 351.7 \times 10^6$ in-lb/rad

$B_1 = B_2 = B_3 = 22.99$ in-lb-sec (1% bearing loss)

$C_1 = 16{,}663$ in-lb-sec, $C_2 = 39{,}411$ in-lb-sec

Example - Synchronous Electric Motor Drive Train

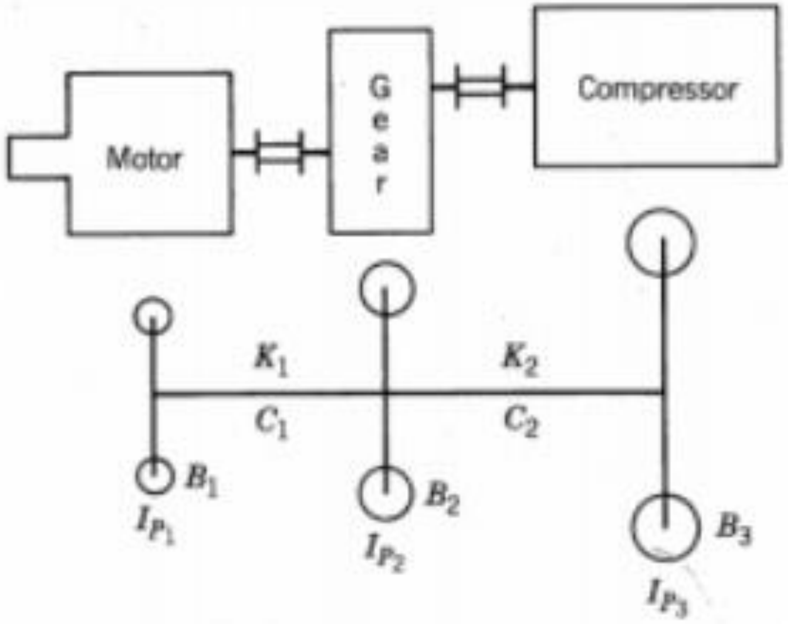


Figure 3.12. Industrial drivetrain with three-inertia torsional model.

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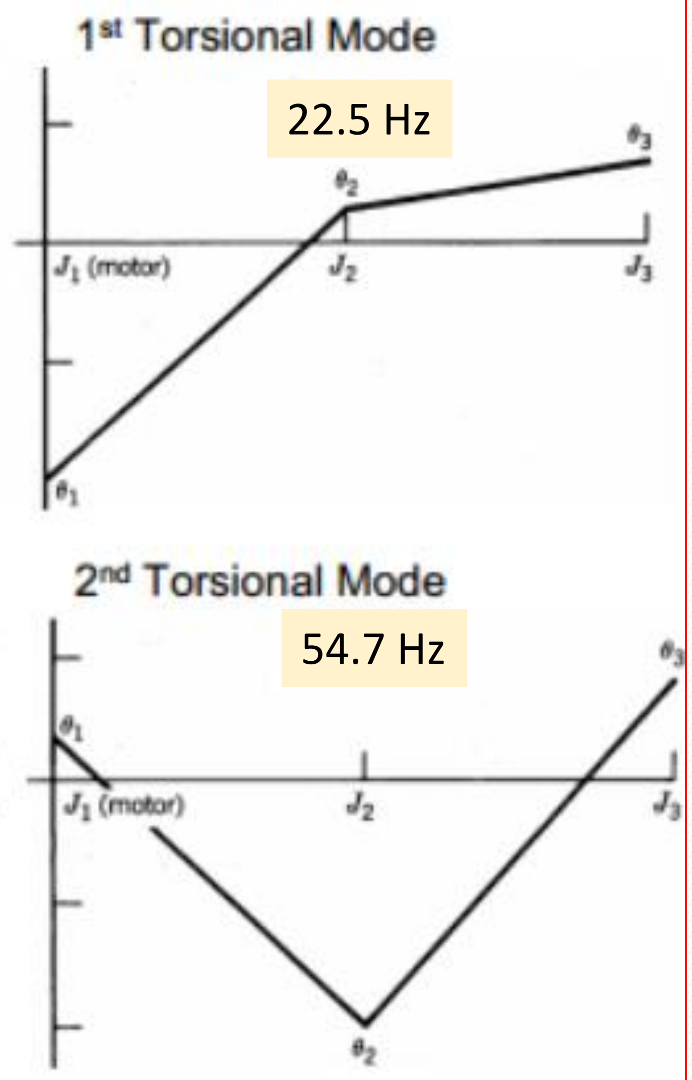


Natural frequencies and mode shapes

Campbell diagram: torsional interference

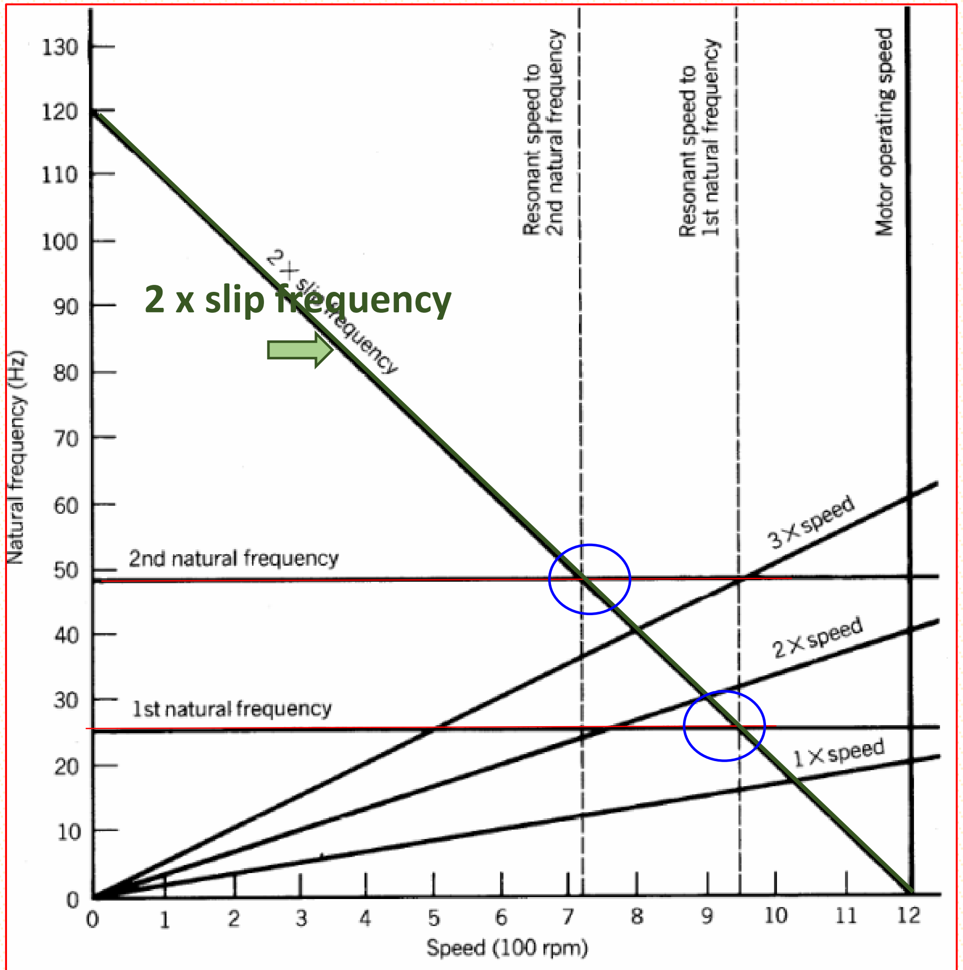


Figure 3.13. Torsional interference diagram.

Motor torque = steady + pulsating

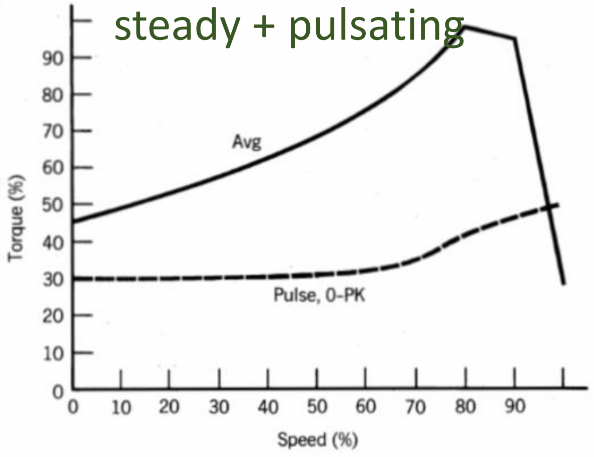


Figure 3.14. Motor torque–speed characteristics.

$$f_e = 2f\left(\frac{N_s - N}{N_s}\right) \text{ Hz,} \qquad \text{(2-14)}$$

f = line frequency, Hz

N_s = synchronous motor speed, rpm

N = actual motor speed, rpm

$$T_1(t) = T_{avg} + T_{osc} \cos\left[4\pi L_f\left(t - \frac{\theta_1}{\Omega_s}\right)\right]$$

T_{avg} = average value of starting torque at time t

T_{osc} = magnitude of the oscillating torque at time t

θ_1 = angular position of the motor inertia at time t

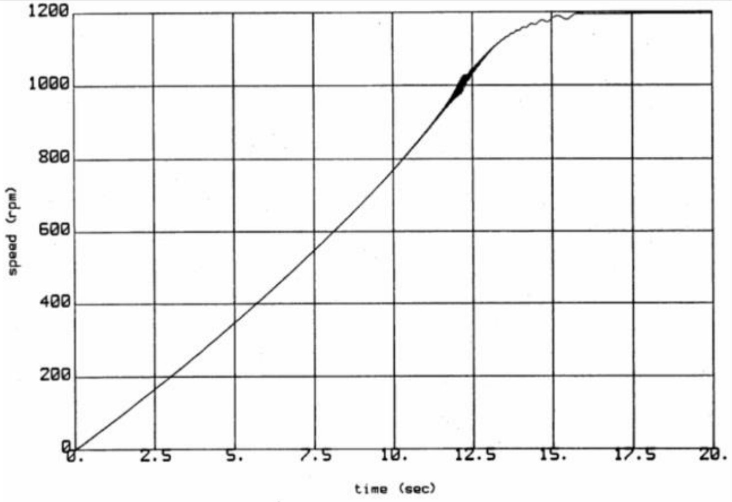


Figure 3.16. Motor speed versus time in start-up of industrial drivetrain.

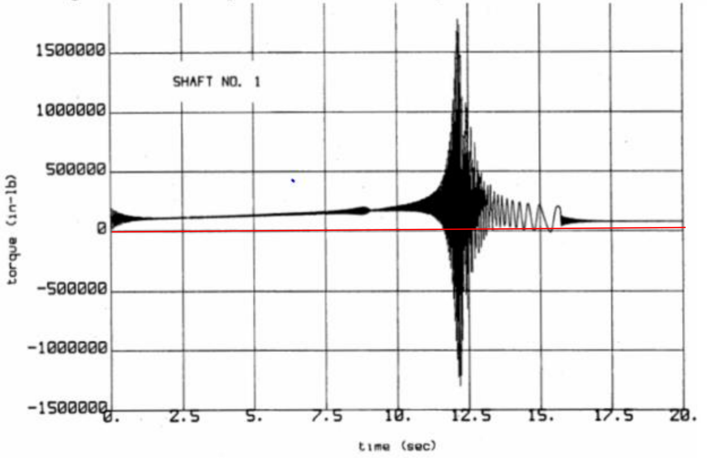


Figure 3.18. Motor shaft torque during start-up.

Typical Torsional Excitations

TABLE 38.4 Sources of Excitation of Torsional Vibration

Source	Amplitude in terms of rated torque	Frequency
Mechanical		
Gear runout		1 ×, 2 ×, 3 × rpm
Gear tooth machining tolerances		No. gear teeth × rpm
Coupling unbalance		1 × rpm
Hooke's joint		2 ×, 4 ×, 6 × rpm
Coupling misalignment		Dependent on drive elements
System function		
Synchronous motor start-up	5–10	2 × slip frequency
Variable-frequency induction motors (six-step adjustable frequency drive)	0.04–1.0	6 ×, 12 ×, 18 × line frequency (LF)
Induction motor start-up	3–10	Air gap induced at 60 Hz
Variable-frequency induction motor (pulse width modulated)	0.01–0.2	5 ×, 7 ×, 9 × LF, etc.
Centrifugal pumps	0.10–0.4	No. vanes × rpm and multiples
Reciprocating pumps		No. plungers × rpm and multiples
Compressors with vaned diffusers	0.03–1.0	No. vanes × rpm
Motor- or turbine-driven systems	0.05–1.0	No. poles or blades × rpm
Engine geared systems with soft coupling	0.15–0.3	Depends on engine design and operating conditions; can be 0.5*n* and *n* × rpm
Engine geared system with stiff coupling	0.50 or more	Depends on engine design and operating conditions
Shaft vibration		*n* × rpm

From [1]

Amplitudes are % of rated torque, 0-pk

Table 5. Steady State Torsional Excitation Sources.

Source	Frequency	Amplitude % T_{SS} 0–p	Comments
Centrifugal Compressors Turbines	1× 2×	1.0% $\frac{1}{3}$	Misalignment
Pumps, Fans	Bx, β × B = Number of Impeller Vanes β = Number of Diffuser Vanes	$\left(\frac{1}{B}\right)$% $\frac{1}{\beta}$%	Assumes No Acoustical Resonances
Gears	1× 2×, 3×, n×	1% $\frac{1}{2}, \frac{1}{3}, \frac{1}{n}$...%	Worn Gears, Bad Allignment
Reciprocating Compressors, Pumps	1×, 2×, n×	TEC * [1-3]	*Torque Effect Curve (TEC) Must Be Developed for Each Cylinder
Lobed Blowers	1×, 2×, n×	10-40% Typical	Harmonic Torques Depend Upon Number of Lobes and Their Timing
Engines (2 Cycle)	1×, 2× ... n×	Harmonic Torque Coefficients [1-3]	Based on Pressure - Time Wave From Power Cylinder Function of Mean Effective Pressure
Engines (4 Cycle)	$\frac{1}{2}$×, 1×, $1\frac{1}{2}$×, 2× $2\frac{1}{2}$× ... n×	Harmonic Torque Coefficients [1-3]	Based on Pressure - Time Wave From Power Cylinder Function of Mean Effective Pressure
Variable Frequency Motors	1×, 2× ... n× 6×, 12× ... 6n×	Mfg. Supplied	Depends Upon Type of VFD

From [2]

[1] Torsional Vibration in Reciprocating and Rotating Machines, Ronald L. Eshleman, Shock and Vibration Handbook, 5th Edition, Harris and Piersol, 2002.
[2] Analysis of Torsional Vibrations in Rotating Machinery, J. C. Wachel and Fred R. Szenasi, 22nd TAMU Turbo Show, 1993. http://turbolab.tamu.edu/proc/turboproc/T22/T22127-151.pdf
[3] Torsional Vibration of Machine Systems, Ronald L. Eshleman, 6th TAMU Turbo Show, 1977. http://turbolab.tamu.edu/proc/turboproc/T6/T6pg13-22.pdf

More Typical Torsional Excitations

Table 1. Summary of Excitation Sources and Frequencies.

Excitation Source	Excitation Frequencies
Generic 1X (unbalance, eccentricity, misalignment, etc.)	One x Speed
Generic 2X (misalignment, ellipticity, etc.)	Two x Speed
Gear Mesh Consisting of Pinion with N_P Teeth Mating with Gear Having N_G Teeth	Pinion Shaft: • One x Pinion Speed • Two x Pinion Speed • N_P x Pinion Speed Gear Shaft: • One x Gear Speed • Two x Gear Speed • N_G x Gear Speed
Impeller with N_R Blades Rotating Inside Casing with N_S Cutwaters	• N_R x Speed • N_S x Speed • n x Speed (n is given by Equation (15))
AC Motor or Generator with N_p Poles (Fixed Frequency or Static Kramer Drive)	• Line Frequency (60 Hz) • Twice Line Frequency (120 Hz) • N_p x Speed
AC Motor with N_p Poles (Variable Frequency Drive Controlling Stator)	• ½ x N_p x Speed • N_p x Speed
Variable Frequency Drive (Stator Frequency Control) with N Pulses Driving AC Motor with N_p Poles	• ½ x N x N_p x Speed • N x N_p x Speed • 1.5 x N x N_p x Speed • 2 x N x N_p x Speed
Static Kramer Drive with N Pulses	• N x Slip Frequency • 2 x N x Slip Frequency
Synchronous Motor (Fixed Frequency Drive)	Two x Slip Frequency

Table 4. Relevant Cases for Various Electrical Machines.

Machine Type	Drive Type	Case	Excitation Frequencies	Steady State or Transient
AC Generator with N_p Poles		1. Steady Running 2. Steady Running 3. Steady Running 4. Short Circuit 5. Short Circuit	Line Frequency 2 x Line Frequency N_p x RPM Line Frequency 2 x Line Frequency	Steady State Steady State Steady State Transient Transient
AC Motor with N_p Poles	Fixed Frequency	1. Steady Running 2. Steady Running 3. Steady Running 4. Short Circuit 5. Short Circuit 6. Initial Start 7. Runup to Speed (synchronous only)	Line Frequency 2 x Line Frequency N_p x RPM Line Frequency 2 x Line Frequency Line Frequency 2 x Slip Frequency	Steady State Steady State Steady State Transient Transient Transient Transient
AC Motor with N_p Poles	Variable Frequency with N Pulses (except Static Kramer)	1. Steady Running 2. Steady Running 3. Steady Running 4. Steady Running 5. Steady Running 6. Steady Running 7. Short Circuit 8. Short Circuit 9. Initial Start (induction only)	½ x N_p x RPM N_p x RPM ½ x N x N_p x RPM N x N_p x RPM 1.5 x N x N_p x RPM 2 x N x N_p x RPM ½ x N_p x RPM N_p x RPM Line Frequency	Steady State Steady State Steady State Steady State Steady State Steady State Transient Transient Transient
AC Motor with N_p Poles	Static Kramer Drive with N Pulses	1. Steady Running 2. Steady Running 3. Steady Running 4. Steady Running 5. Steady Running 6. Short Circuit 7. Short Circuit 8. Initial Start	Line Frequency 2 x Line Frequency N_p x RPM N x Slip Frequency 2N x Slip Frequency Line Frequency 2 x Line Frequency Line Frequency	Steady State Steady State Steady State Steady State Steady State Transient Transient Transient

[*] Practical Design Against Torsional Vibration, Mark A. Corbo and Stanley B. Malanoski, 25th TAMU Turbo Show, 1996. http://turbolab.tamu.edu/proc/turboproc/T25/T25189-222.pdf

VFDs → Torsional Excitations

- VFD's, in addition to well known harmonic distortion components at 6x, 12x, etc., also possess inter-harmonic distortion which is the difference between line frequency and drive output frequency, f_l and f_o (API 684; 4.5.1.5)

$$f_{exc} = (kf_o \pm mf_l)$$

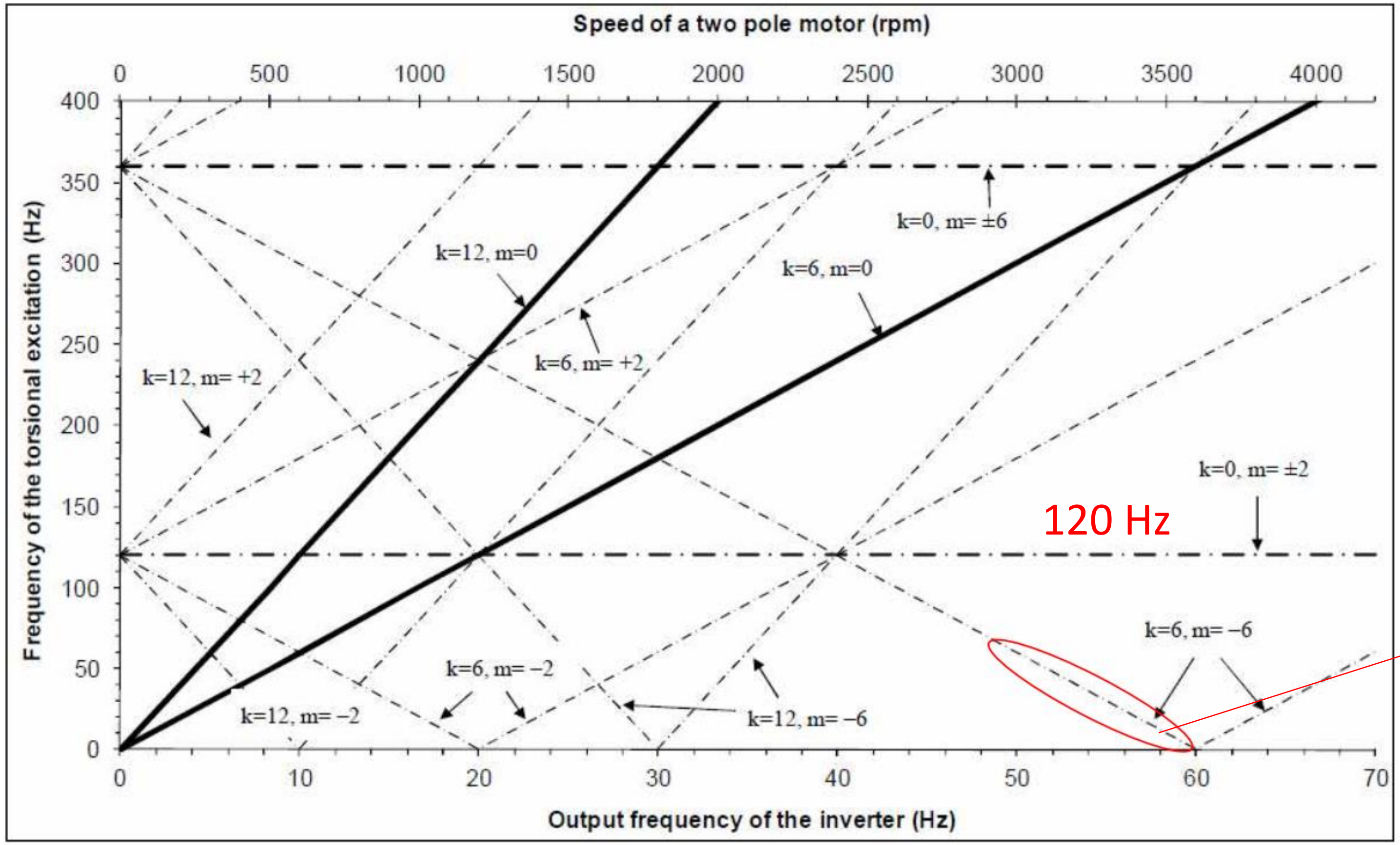
- The most important of which have $k=m=6$ or $k=m=12$ and ± is usually "-", but "+" could be important if the unit is operated with $f_o > f_l$
- In large compressor strings, excitation of the 1st torsional mode is of greatest concern because of its low frequency and low torsional damping (e.g. 15 hz and $\xi < 1\%$)
- At train startup f_{exc} begins at high frequency, and drops to zero at 100% speed, so the resonant critical speed may be not far below MCOS and inside the operating speed range

MCOS: maximum continuous operating speed

Torsional Excitations (cont.)

- Graph of VFD torsional excitations $f_{exc} = (kf_o \pm mf_l)$

frequencies
f_l: line frequency
f_o: operating frequency

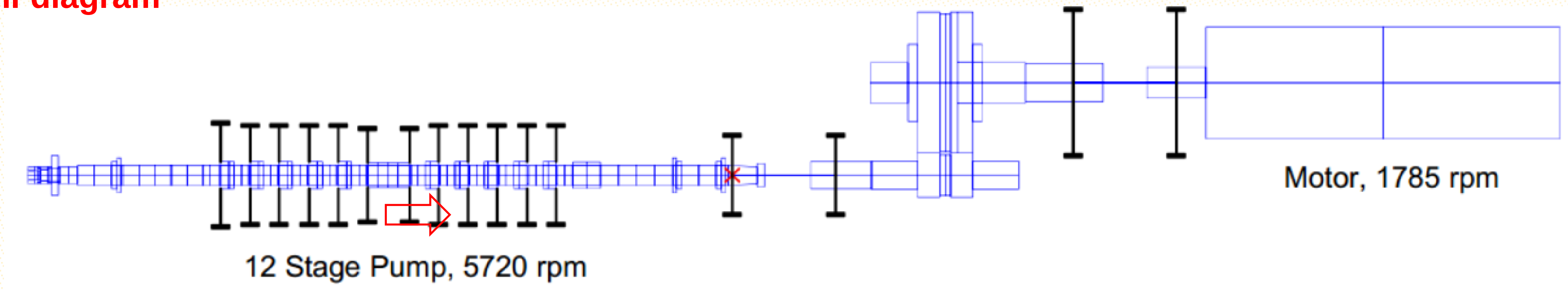


Inter-harmonic excitation 6 x (f_o-f_l) that can resonate the 1st torsional mode at speeds near the top of the operating speed range, where drive and load torques are highest.

[*] 2013 Electric Motors and Drives in Torsional Vibration Analysis and Design; Timo P. Holopainen, PiederJörg, Jouko Niiranen, Davide Andreo

Torsional Eigenanalysis for Critical Speeds

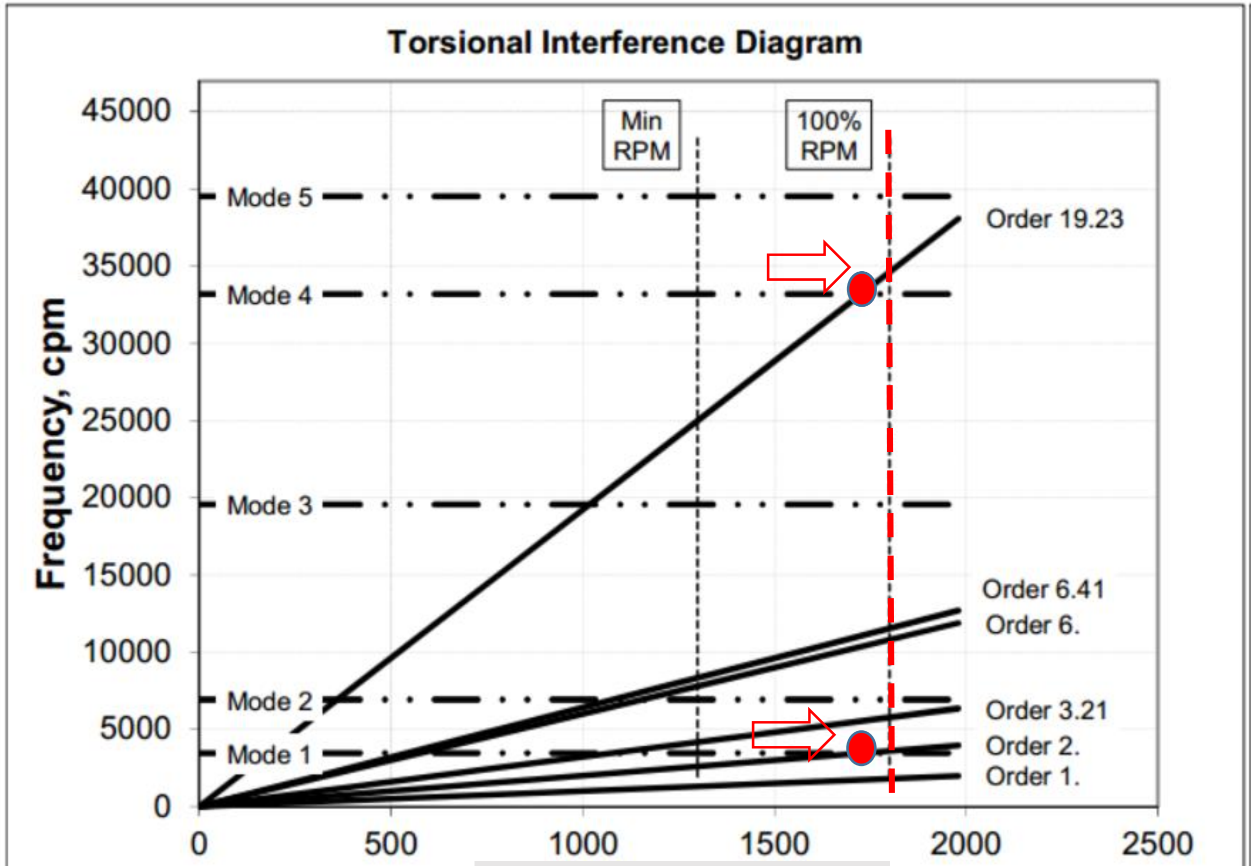
- **Torsional natural frequency calculation**
  – Campbell diagram



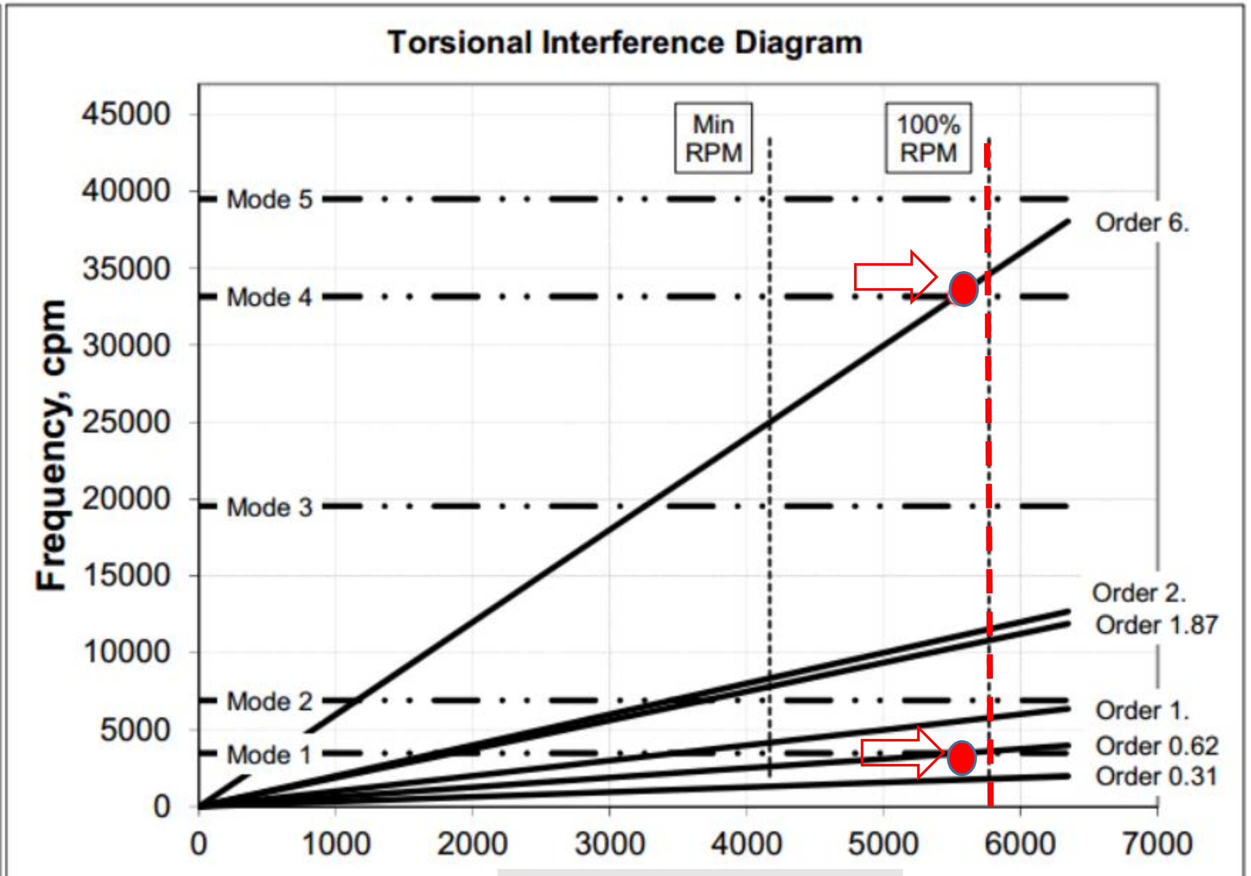
- 12 stage pump, 6 vane impellers, 6 pulse VFD, 3.205 gearbox, operating speed 1300 rpm to 1800 rpm (4166 to 5769 rpm@pump)
- **The potential torsional excitations to consider for this train are:**
  - 1X and 2X of the motor
  - 1X and 2X of the pump (the gear ratio is 3.205), which are the same as 3.205X and 6.410X of the motor
  - Vane pass, so 6X of the pump or 19.23X of the motor
  - A VFD frequency of 6X of the motor. The VFD maker says 12X and higher are negligible (note this is nearly the same as 2X of the pump)

Critical Speeds & Interference Diagrams

T wo torsional interference diagrams: natural frequencies vs motor & vs pump speed (3.205 gearbox)



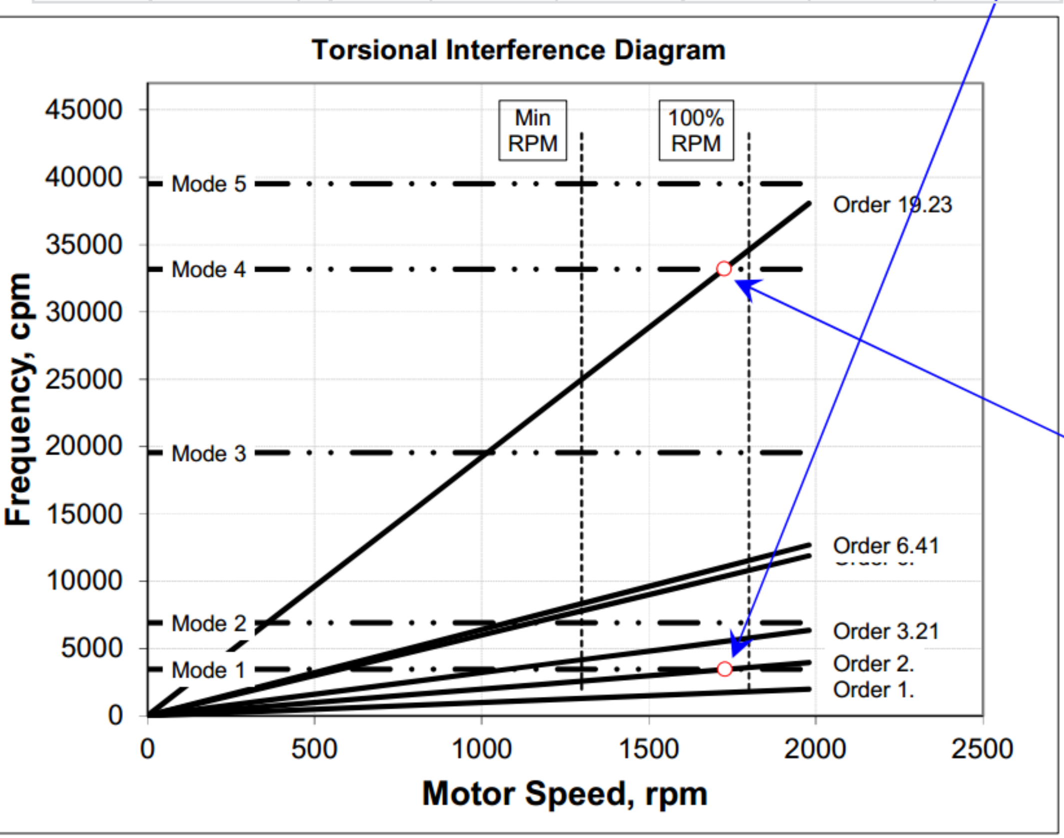
Motor speed (rpm)

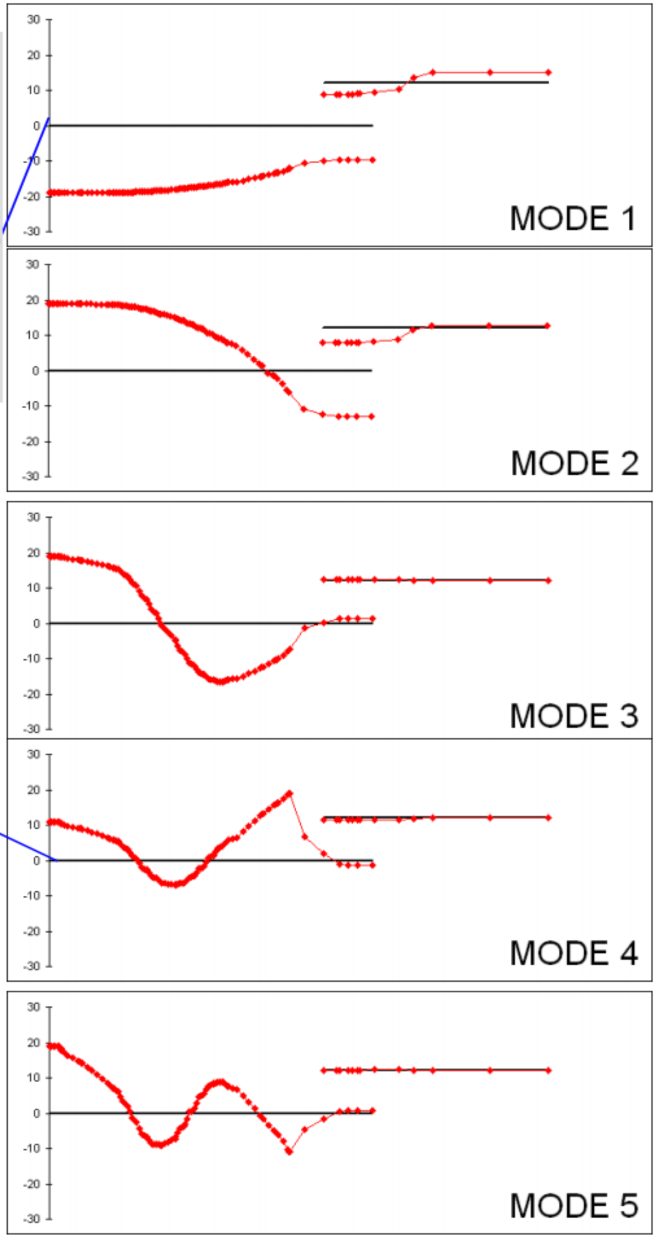


Pump speed (rpm)

Modes

		Motor Order Critical Speeds in rpm, c.s. margin=±0.1					
		Order	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
	cpm		3456.3	6913.8	19551.4	33188.5	39530.8
Mode 1	3456.292	1					
Mode 2	6913.76	2	**1728.1**				
Mode 3	19551.36	3.205					
Mode 4	33188.5	6					
Mode 5	39530.75	6.41					
		19.23				**1725.9**	





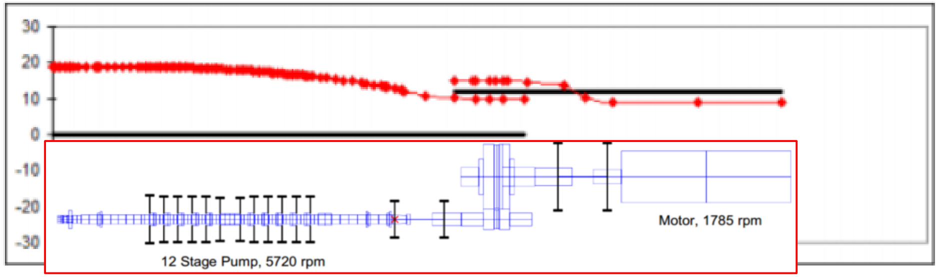
- Modes higher than the 5th are out of the range of the excitation frequencies
- The table of critical speeds lists all values of motor speed in the operating speed range ±10% (1170 to 1980 rpm) where a natural frequency equals an excitation frequency

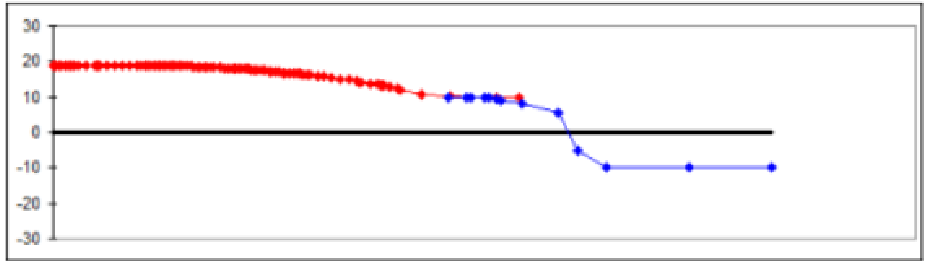
Torsional Modes Shape Display

- *Physical* shaft output
  - 2 Rotors are offset
  - Magnitudes include gear ratio

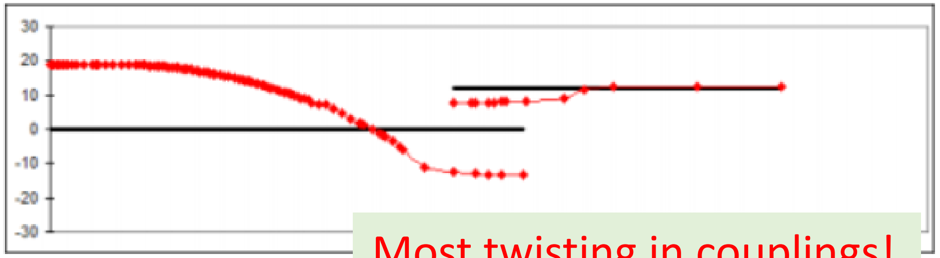
- *Equivalent* shaft output
  - Rotors on common CL
  - Referenced to same speed

Mode 1

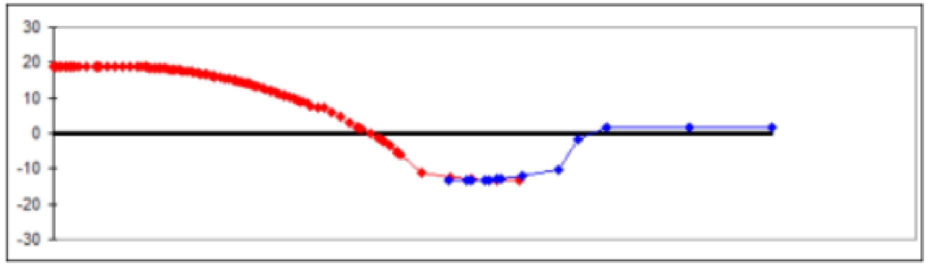




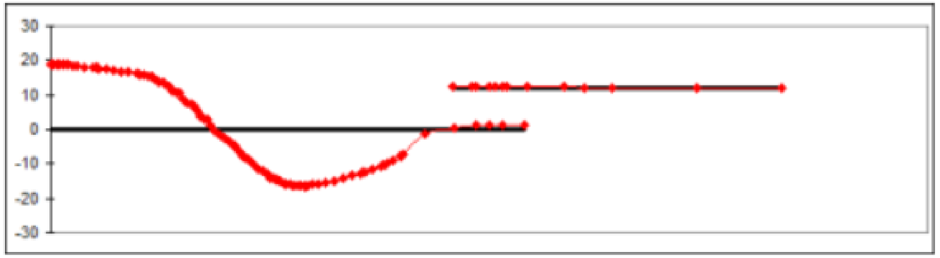
Mode 2

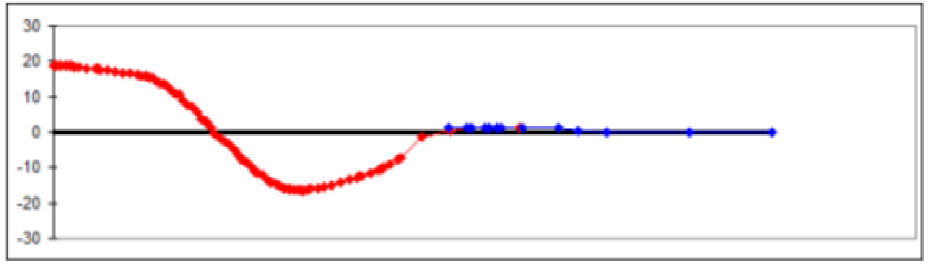


Most twisting in couplings!



Mode 3





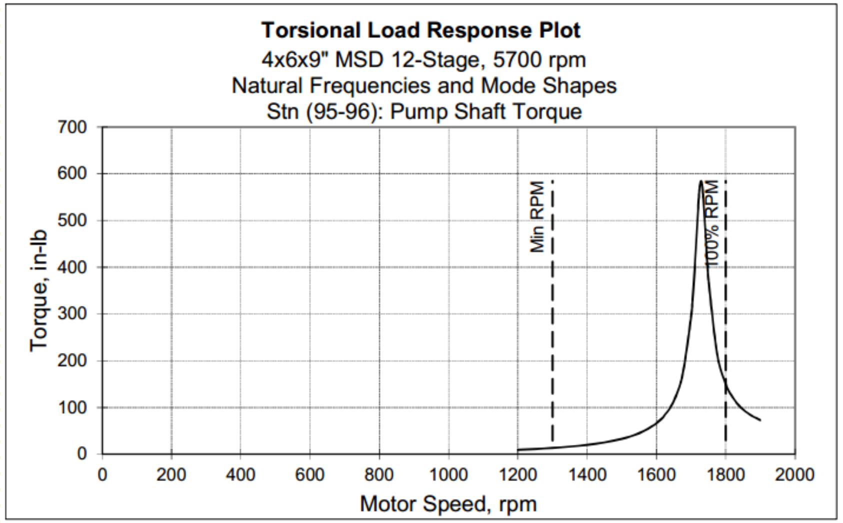
Torsional Forced Response

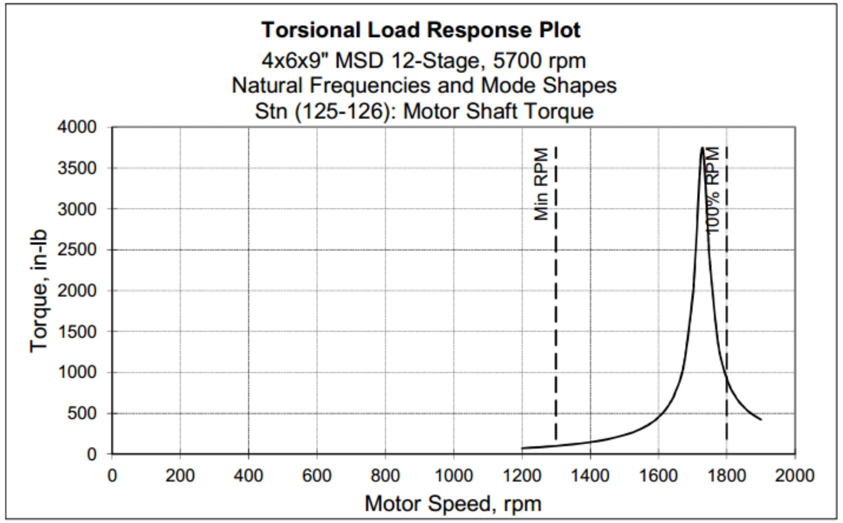
- The interference of 2x motor speed with the first mode when speed=1728 rpm can be evaluated with a response analysis.
- The motor is 550 hp at 1800 rpm, which equates to 19,300 in-lb nominal motor drive torque.
- We will apply 1% of this torque at the motor, at a frequency of 2x motor speed
- We will put damping into the model so that the damping ratio of the first mode is 1%. This should be conservative as actual damping in pumps ought to be higher.

Adding Proportional Damping to the Model

- "Stiffness proportional damping" is a damping matrix proportional to the stiffness matrix.
- In a linear forced response analysis, a damping matrix of $C=K*(2\zeta/\omega)$ where
  - K is the system stiffness matrix
  - ζ is the desired damping ratio
  - ω is the frequency of vibration
- Will produce the desired damping ratio ζ for a response calculation done at a frequency of ω.

Results





- The critical speed is 1725 rpm where it should be.
- The max torque in the motor shaft is 3750 in-lb pk (19.4% of nominal drive torque)
- A thorough evaluation of shaft stress would be required to decide if this is too high to run the pump on this critical
- **If fatigue life were not infinite, 1725±173 rpm should be excluded as an operating point.**
- Other things that might help are to lessen conservatism in the analysis:
  - Ask motor manufacturer for a value for the 2x pulsating torque magnitude, it should be <1%
  - Apply actual damping of the pump impellers and bearings, this should increase the damping ratio.

The twisted road ahead

Do learn more…… There are many articles/lectures/tutorials presented at the Turbomachinery & Pump Symposium.

Excitation of torsional frequencies with large shaft angular motions (even failure) is not uncommon as VFDs become larger and larger (in power).

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notes